A unified account of the distributive and free choice inferences of disjunction under modals

Disjunction embedded into epistemic modal expressions like *possible*, *likely*, and *certain* give rise to the inferences that the disjuncts are epistemically possible. While identical, these inferences are classified and treated differently, with the ones in (1a/b) labeled ‘free choice’ inferences while those in (2a/b), ‘distributive’ ones. Distributive inferences are routine predictions of most standard accounts of implicature (Sauerland 2004, Fox 2007, a.o.), while free choice inferences are notoriously problematic (Kratzer and Shimoyama 2002, Fox 2007, Chierchia 2013 a.o.). We show that a degree-based semantics for modals can easily predict all these inferences via the same mechanism. Our proposal dovetails well with (though does not require endorsing) recent probability-based semantics for epistemic modals (Yalcin 2010, Lassiter 2011, 2014 a.o.), and hence can be seen as an indirect argument for analyses in this vein. On the other hand, we show that our analysis can also be implemented by accommodating Klecha’s 2014 worries against probabilistic theories of these modals. We conclude by showing how the proposal can be extended to deontic modals, given reasonable assumptions about a degree semantics for these modals.

(1) It’s possible that it will rain or snow	(2) It’s likely/certain that it will rain or snow
(a) $\rightsquigarrow$*it is possible that it will rain*	(a) $\rightsquigarrow$*it is possible that it will rain*
(b) $\rightsquigarrow$*it is possible that it will snow*	(b) $\rightsquigarrow$*it is possible that it will rain*

Degree semantics for modal adjectives. We adopt a standard analysis of degree adjectives (Heim 2000 a.o.) and assume that the meaning of *possible/likely/certain* is that in (4), mapping a proposition to the set of degrees up to which that proposition is probable.[1] In addition, we make the standard assumption that in the positive form these adjectives combine with a silent morpheme POS, which contributes an operation of existential closure binding the degree variable as in (5), where **s** is the standard function (Kennedy & McNally 2005, Kennedy 2007 a.o.)

(4) $[\![\text{possible/certain/likely}]\!] = \lambda p \lambda d[Pr(p) \geq d]$

(5) $[\![\text{POS}]\!] = \lambda G_{\langle d,t \rangle} \exists d[\mathbf{s}(G)(d) \wedge G(d)]$

The standard function **s** gives different results depending on the type of adjectives it combines with. In particular, the result of combining POS with absolute adjectives like *possible* and *certain* is that the probability of raining is not 0 and equals to 1, respectively. In the case of a relative adjective like *likely*, the standard varies contextually (cf. Yalcin 2010, Lassiter 2011, 2014).

(6) $[\![\text{it is POS [possible [that it is raining]]}]\!] = \exists d[d > 0 \wedge Pr(\mathbf{rain}) \geq d]$

(7) $[\![\text{it is POS [likely [that it is raining]]}]\!] = \exists d[d > \mathbf{s}(\mathbf{likely}) \wedge Pr(\mathbf{rain}) \geq d]$

(8) $[\![\text{it is POS [certain [that it is raining]]}]\!] = \exists d[d = 1 \wedge Pr(\mathbf{rain}) \geq d]$

Scalar implicatures. For concreteness, we adopt an exhaustification-based account of scalar implicatures (Chierchia et al. 2012, Fox 2007 a.o.). Scalar implicatures are generated by an exhaustivity operator EXH. EXH takes as arguments a sentence S and a set of alternatives $\mathcal{A}lt(S)$, and returns the conjunction of S with the negation of the ‘excludable’ alternatives in $\mathcal{A}lt(S)$. An alternative is excludable just in case (a) negating it doesn’t contradict the assertion S, and (b) negating it doesn’t force us to accept any other alternative in $\mathcal{A}lt(S)$ (Sauerland 2004, Fox 2007).

[1] More precisely, given any propositional argument ϕ, we write (i): where e is a relevant epistemic state; epistemic states are pairs $\langle E, Pr \rangle$ of a set of possible worlds E and a probability measure Pr defined over E.

(i) $[\![\text{possible/certain/likely } \phi]\!]^{e,w} = \lambda d[Pr_{e,w}(\{w' : [\![\phi]\!]^{e,w'}\}) > d]$

(8) $[\![\textsf{EXH}[\phi][\mathcal{A}lt(\phi)]]\!]^w = [\![\phi]\!]^w \wedge \forall\psi \in \textsf{excl}(\phi, \mathcal{A}lt(\phi))[\neg[\![\psi]\!]^w]$
(i) $\textsf{excl}(\phi, \mathcal{A}lt(\phi)) = \{\psi \in \mathcal{A}lt(\phi) : \phi \not\subseteq \psi \wedge \neg\exists\chi[\chi \in \mathcal{A}lt(\phi) \wedge (\phi \wedge \neg\psi) \subseteq \chi]\}$

Deriving the inferences The inferences of *certain* follow straightforwardly from exhaustification and standard assumptions about alternatives. The exhaustification of (2a) (with *certain*) involves negating the alternatives in (9) giving rise to the strengthened meaning in (10). Informally, exhaustification has the effect of 'distributing' the probability over the two disjuncts: the strengthened meaning of (2) says that the probability of rain or snow is 1 and that the probabilities of each of rain and snow is smaller than 1. Given standard assumptions about probability measures, it follows that the probabilities of both rain and snow are positive. But then, given our analysis of *possible* in (6), this, in turn, entails the inferences in (2a) and (2b).

(9) {it is certain that it will rain, it is certain that it will snow}
(10) ⟦it is EXH[POS certain that it will rain or snow]⟧ =
$Pr(r \vee s) = 1 \wedge \neg(Pr(r) = 1) \wedge \neg(Pr(s) = 1) \quad \Rightarrow \quad Pr(r) > 0 \wedge Pr(s) > 0$

The inferences with *likely* are easily accounted for in a completely analogous way. Assuming for concreteness that **s(likely)** is .5 in the context, the alternatives negated are in (11) and the strengthened meaning is (12), which entails the inferences in (2a) and (2b), in the same way as above.

(11) {it is likely that it will rain, it is likely that it will snow}
(12) $Pr(r \vee s) > .5 \wedge \neg(Pr(r) > .5) \wedge \neg(Pr(s) > .5) \quad \Rightarrow \quad Pr(r) > 0 \wedge Pr(s) > 0$

Finally, the inferences of *possible* in (1a) and (1b) are also captured in the same way. The only extra assumption required is that the relevant exhaustivity operator scopes between POS and the modal (as in (13)). Analogous assumptions about alternatives yield that the meaning of the constituent headed by EXH (before combing with POS) is (14):

(13) ⟦it is POS [EXH [possible that it will rain or snow]]⟧
(14) ⟦EXH possible that it will rain or snow⟧ = $Pr(r \vee s) > d \ \wedge \ Pr(r) \leq d \ \wedge \ Pr(s) \leq d$

Applying POS we get (15), which entails (16). But then, again, on our semantics, (16) amounts precisely to the assertion conjoined with its inferences in (1a) and (1b).

(15) $\exists d\, [d > 0 \wedge Pr(r \vee s) > d \ \wedge \ Pr(r) \leq d \ \wedge \ Pr(s) \leq d]$
(16) $\exists d[d > 0 \wedge Pr(r \vee s) > d] \ \wedge \ \exists d[Pr(r) > d] \ \wedge \ \exists d[Pr(s) > d]$

In sum: a degree-based semantics for modal adjectives like *possible*, *certain* and *likely* provides a unified account of the inferences in (1a) and (1b) and the similar inferences in (2a) and (2b) as a simple scalar implicature.

Extensions: Free choice inferences are also triggered by other modals, in particular deontic modals. Our account can be extended to these cases, provided that the relevant operators is given a semantics based on a degree measure that vindicates the property in (16) (Holliday and Icard 2013, Cariani 2013, Wedgwood 2015).
(16) ADDITIVITY *Degree*$(p) \geq$ *Degree*(q) iff *Degree*$(p \wedge \neg q) \geq$ *Degree*$(q \wedge \neg p)$

Selected References: • **Fox, D. 2007**. *Free choice and the theory of scalar implicatures*. • **Holliday, W. and Icard, T. 2013**. *Measure semantics and qualitative semantics for epistemic modals*. • **Lassiter, D.: 2011**. *Measurement and Modality*. • **Yalcin, S.: 2010**. *Probability operators*.