1 Introduction

- Consider the following sentence:

(1) John knows who called.

- Assume a domain of discourse $D$, and assume that the set of answers to (1) is:

$$A = \{d \text{ called} \mid d \in D\}$$

- Traditionally, three readings for sentences like (1) are distinguished:

**Strongly exhaustive reading:**
- for any true answer $a \in A$, John knows that $a$ is true, and
- for any false answer $a \in A$, John knows that $a$ is false

**Weakly exhaustive reading:**
- for any true answer $a \in A$, John knows that $a$ is true

**Mention-some reading:**
- for at least one true answer $a \in A$, John knows that $a$ is true

- Note that false answers only play a role for strongly exhaustive readings in this tradition.

- Recently, they have been shown to play a role for weaker levels of exhaustivity as well.

- Spector’s (2005) observation, somewhat adapted:
  - Consider the following scenario:

<table>
<thead>
<tr>
<th>Who passed the exam?</th>
<th>Ann</th>
<th>Bill</th>
<th>Chris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>What John told Mary</td>
<td>Y</td>
<td>Y</td>
<td>-</td>
</tr>
<tr>
<td>What Peter told Mary</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>
– Ann and Bill were the only people who passed the exam.
– John told Mary that Ann and Bill passed, and he didn’t tell her anything else.
– Peter told Mary that Ann, Bill and Chris passed.
– Then:

(2) John told Mary who passed the exam. ⇒ true (weakly exhaustive)
(3) Peter told Mary who passed the exam. ⇒ false

– Apparently, it matters that Peter did not only tell Mary all the true answers to the embedded interrogative, but also a false answer.

• George’s (2013) observation, again somewhat adapted:

– Scenario:

<table>
<thead>
<tr>
<th>Where can one buy an Italian newspaper?</th>
<th>Newstopia</th>
<th>Cellulose City</th>
<th>Paperworld</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>What Janna believes</td>
<td>Y</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>What Rupert believes</td>
<td>Y</td>
<td>-</td>
<td>Y</td>
</tr>
</tbody>
</table>

– Italian newspapers are only sold at Newstopia and Cellulose City.
– Janna knows that Italian newspapers are sold at Newstopia, and does not have beliefs concerning the availability of Italian newspapers elsewhere.
– Rupert knows that Italian newspaper are sold at Newstopia, but he also falsely believes that Italian newspapers are sold at Paperworld.
– Then (4) is judged true on a mention-some reading, while (5) is judged false.

(4) Janna knows where one can buy an Italian newspaper. ⇒ true (mention-some)
(5) Rupert knows where one can buy an Italian newspaper. ⇒ false

– Apparently, it matters that Rupert does not only know a true answer to the embedded interrogative, but also wrongly believes a false answer.

• Thus, it seems that the traditional characterization of weakly exhaustive and mention-some readings needs to be adapted. For our initial example (1) we get the following:

**Strongly exhaustive reading, as before:**
– for any true answer \(a \in A\), John knows that \(a\) is true, and
– for any false answer \(a \in A\), John knows that \(a\) is false

**FA sensitive weakly exhaustive reading:** (aka intermediate exhaustive reading)
– for any true answer \(a \in A\), John knows that \(a\) is true
– for any false answer \(a \in A\), John does not believe that \(a\) is true
FA sensitive mention-some reading:
- for at least one true answer \(a \in A\), John knows that \(a\) is true
- for any false answer \(a \in A\), John does not believe that \(a\) is true

- Recent accounts:
  - either focus on **FA sensitive mention-some** readings (George, 2011, 2013)
  - or on **FA sensitive weakly exhaustive**, aka **intermediate exhaustive** readings
    (Spector, 2005; Klinedinst and Rothschild, 2011; Égré and Spector, 2014; Uegaki, 2014)

- Our goals today:
  - Give a **general** characterization of FA sensitivity that applies uniformly across different levels of exhaustive strength.
  - Specify a **modular** compositional derivation of the different readings that clearly brings out at which point FA sensitivity comes in and how it interacts with other parameters.
  - Show how the account of embedded interrogatives can be extended to one that deals **uniformly** with embedded declaratives and interrogatives.
  - Show how the account can be extended to cases where the embedding verb is not an **extensional** verb like **know**, but an **intensional** verb like **wonder** or **be certain**.

2 Disentangling exhaustivity and completeness

- In order to give a general characterization of FA sensitivity and its interaction with other parameters, it is crucial to **decompose** the traditional **three-way distinction** between weak/strong exhaustivity and mention-some into two **two-way distinctions**.

- One of these concerns **exhaustivity**, the other **completeness**.

- For concreteness, assume that an embedded interrogative clause has the following structure:

```
embedded interrogative clause
  \(A_{[+/-cmp]}\)  \(\uparrow\)  \(\uparrow\)  \(\uparrow\)
  \(\downarrow\)  \(\downarrow\)  \(\downarrow\)
  answers  \(\uparrow\)  \(\uparrow\)
  \(\uparrow\)  \(\downarrow\)
  \(?_{[+/-exh]}\)  \(\downarrow\)
  resolutions  \(\downarrow\)
  prejacent  \(n\)-place property
```

- Let’s consider the prejacent, the \(?\) operator, and the \(A\) operator in turn.
2.1 The prejacent

- We assume that the prejacent expresses an \( n \)-place property.
  (cf. Hull, 1975; Hausser and Zaefferer, 1978; Groenendijk and Stokhof, 1984; Krifka, 2001)

- More precisely, we take the prejacent to be of type \( \langle e^n, T \rangle \), where:
  - \( n \) is the number of \( \text{wh} \)-elements (\( n = 0 \) in the case of a polar interrogative)
  - \( T \) abbreviates \( \langle \langle s, t \rangle, t \rangle \): the type of root clauses in alternative/inquisitive semantics
    (cf. Hamblin, 1973; Kratzer and Shimoyama, 2002; Ciardelli and Roelofsen, 2014a; Theiler, 2014)
  - For instance:
    - \( \text{whether Bill called} \Rightarrow \text{type } T \)
    - \( \text{who called} \Rightarrow \text{type } \langle e, T \rangle \)
    - \( \text{who ate what} \Rightarrow \text{type } \langle e, \langle e, T \rangle \rangle \)

- We assume the following kind of translations:
  - \( \text{whether Bill called} \Rightarrow \lambda p. \langle s, t \rangle. \forall w \in p : \text{called}(w)(b) \)
  - \( \text{who called} \Rightarrow \lambda x. \lambda p. \langle s, t \rangle. \forall w \in p : \text{called}(w)(x) \)
  - \( \text{who ate what} \Rightarrow \lambda y. \lambda x. \lambda p. \langle s, t \rangle. \forall w \in p : \text{ate}(w)(x)(y) \)

- These are functions that take \( n \geq 0 \) individuals as input and deliver a set of propositions.

2.2 The ? operator

- Informal characterization of the ? operator
  The \( ? \) operator yields, given the \( n \)-place property expressed by its prejacent \( \varphi \) and a domain of discourse \( D \), the set consisting of all propositions that (exhaustively) specify the property expressed by \( \varphi \).

- Definition: specifying a property
  Given a domain of discourse \( D \), we say that:
  - a proposition \( \text{specifies} \) a property \( R \) iff it conveys of at least one \( d \in D \) that it satisfies \( R \), or it conveys that no \( d \in D \) satisfies \( R \).
  - a proposition \( \text{exhaustively specifies} \) a property \( R \) iff it determines for each \( d \in D \) whether it satisfies \( R \) or not.

- Note: whether a proposition (exhaustively) specifies a property \( R \) is \textbf{world-independent}.

- Formal characterization of the ? operator

  \[
  ?^n \text{[−exh]} := \lambda R. \langle e^n, T \rangle. \lambda p. (\exists \vec{x}. R(\vec{x})(p) \land \forall q \subseteq p. \neg \exists \vec{x}. R(\vec{x})(q))
  \]

  \[
  ?^n \text{ [+exh]} := \lambda R. \langle e^n, T \rangle. \forall \vec{x}. (R(\vec{x})(p) \land \forall q \subseteq p. \neg R(\vec{x})(q))
  \]

- Note that the ? operator has a parameter, \( n \), corresponding to the arity of its prejacent.
• In case \( n = 0 \), the exhaustive and the non-exhaustive variant coincide:

\[
?^0_{[+\text{exh}]} = ?^0_{[-\text{exh}]} = \lambda R_T. \lambda p(s,t). (R(p) \lor \forall q \subseteq p. \neg R(q))
\]

This is precisely what we want for polar interrogatives (see examples below).

• Since the two coincide, we will simply write \( ?^0 \) instead of \( ?^0_{[+\text{exh}]} \) or \( ?^0_{[-\text{exh}]} \).

• Note: \( ?^0 \) behaves exactly like the non-informative projection operator \( ? \) in basic inquisitive semantics, where everything is purely propositional.

• \( ?^n_{[+\text{exh}]} \) and \( ?^n_{[-\text{exh}]} \) embody different ways of generalizing this operator to apply to \( n \)-place properties, rather than just to 0-place properties (here, sets of propositions).

• The semantic value of an interrogative clause \( \psi = [?_{[+/-\text{exh}]} \varphi] \) is a set of propositions, which can be thought of as those propositions that resolve the issue that \( \psi \) expresses.

• Downward closure

  – It can be shown that this set of propositions, \( [\psi] \), is always downward closed:

    \[
p \in [\psi] \text{ and } q \subset p \implies q \in [\psi]
    \]

  – This captures the fact that, if \( p \) resolves the issue expressed by \( \psi \), and \( q \) is more informative than \( p \), then \( q \) cannot fail to resolve the issue expressed by \( \psi \) as well.\(^1\)

• Alternatives

  – Among the propositions that resolve \( \psi \), those that are minimally informative have a special status; they are in a sense the most basic resolving propositions.

  – We refer to these propositions as the alternatives in \( [\psi] \).

  – For any set of propositions \( S \), we write \( \text{ALT}(S) \) for the set of minimally informative propositions in \( S \):

    \[
    \text{ALT}(S) := \{ p \in S \mid \neg \exists q \in S, p \subset q \}
    \]

  – For simplicity, we assume throughout the talk that for every clause \( \psi \), \( [\psi] \) can be retrieved from \( \text{ALT}( [\psi] ) \) by just taking all the subsets of the given alternatives.

  – Our definitions could in principle be ‘lifted’ to avoid this assumption, but this would involve some complexities that would obscure the main conceptual points.

(see Ciardelli, 2010, 2014; Theiler, 2014, for discussion)

\(^1\)Detailed motivation for capturing inquisitive content by means of downward closed sets of propositions is given in inquisitive semantics (Ciardelli et al., 2012, 2013). By contrast, classical proposition-set theories of questions (Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984) do not assume downward closure. See Roelofsen (2013), Ciardelli (2014), and Ciardelli and Roelofsen (2014a) for comparison and discussion.
• Examples

\begin{tabular}{|c|c|}
\hline
?_{−\text{exh}} who called & ?_{+\text{exh}} who called \\
\hline
11 & 11 \\
10 & 10 \\
01 & 01 \\
00 & 00 \\
\hline
\end{tabular}

\begin{tabular}{|c|c|}
\hline
?_{0} whether Ann called & \\
\hline
11 & 11 \\
10 & 10 \\
01 & 01 \\
00 & 00 \\
\hline
\end{tabular}

• In these Figures:
  
  – 11 is a world where both Ann and Bill called
  – 10 is a world where only Ann called
  – 01 is a world where only Bill called
  – 00 is a world where neither Ann nor Bill called

• We have only depicted the alternatives — by downward closure, every proposition that is included in one of these alternatives is also part of the semantic value of the clause.

2.3 The $A$ operator

• We take the $A$ operator to be present in embedded clauses, declarative or interrogative.

• We focus first on the interrogative case, but will return to the declarative case later.

• Informal characterization of the $A$ operator

  The $A$ operator yields, given the semantic value of an embedded clause $\psi$, a function that maps every world $w$ to the set of (complete) suitable answers to $\psi$ in $w$.

• It will become clear presently why we use the term suitable here rather than true.

• Definition: answerhood

  – A proposition is a complete suitable answer to a clause $\psi$ in a world $w$ iff it confirms every alternative in $\text{ALT}(\psi)$ that is true in $w$.

  – A proposition is a partial suitable answer to a clause $\psi$ in a world $w$ iff it confirms at least one but not every alternative in $\text{ALT}(\psi)$ that is true in $w$.

  – A proposition is a suitable answer to $\psi$ in $w$ iff it is either a complete or a partial suitable answer to $\psi$ in $w$.

• Note that our notion of suitable answers is world-dependent, unlike our notion of resolving propositions and alternatives.

  – For instance, $\text{Bill called}$ is a suitable answer to $?_{0}\text{whether Bill called}$ in a world where Bill indeed called, but not in a world where Bill did not call.

• Note also that a suitable answer does not need to be true.
For instance, *Bill and Mary called* is a suitable answer to \( \Box \) \( \text{whether Bill called} \) in a world where Bill indeed called, even if Mary didn’t call.

- **Formal characterization of the \( A \) operator**

  \[
  A_{[-\text{cmp}]} := \lambda S. \lambda w. \lambda p(s,t). \exists q \in \text{ALT}(S). (q(w) \land p \subseteq q)
  \]

  \[
  A_{+[\text{cmp}]} := \lambda S. \lambda w. \lambda p(s,t). \forall q \in \text{ALT}(S). (q(w) \rightarrow p \subseteq q)
  \]

- **Examples**

  In each case we depict the **minimally informative** suitable answers in each world:

  \[
  A_{[-\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

  \[
  A_{[+\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

  \[
  A_{[-\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

  \[
  A_{[+\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

  \[
  A_{[-\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

  \[
  A_{[+\text{cmp}]}(\begin{array}{c}
  w_1 \mapsto \begin{array}{c}
  \Box \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_2 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_3 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array} \\
  w_4 \mapsto \begin{array}{c}
  \circ \circ \\
  \circ \circ \\
  \circ \circ \\
  \circ \circ 
  \end{array}
  \end{array})
  \]

- **Downward closure**

  - Suppose \( p \) is a suitable answer to \( \psi \) in \( w \).
  - Then \( p \) confirms at least one alternative in \( \text{ALT}(\{\psi\}) \) that is true in \( w \).
  - Now suppose \( q \subseteq p \).
  - Then \( q \) also confirms at least one alternative in \( \text{ALT}(\{\psi\}) \) that is true in \( w \).
  - This means that \( q \) is also a suitable answer to \( \psi \) in \( w \).
  - So the set of suitable answers to \( \psi \) in \( w \) is always **downward closed**, just like the set of propositions that resolve the issue expressed by \( \psi \).
  - The same goes for **complete** suitable answers.
2.4 Back to strong/weak exhaustivity and mention-some

- The traditional three-way distinction can now be captured as follows:
  - mention some = \([A_{[-cmp]} \ ?_{[-exh]} \varphi]\)
  - weakly exhaustive = \([A_{[+cmp]} \ ?_{[-exh]} \varphi]\)
  - strongly exhaustive = \([A_{[+cmp]} \ ?_{[+exh]} \varphi]\) or \([A_{[-cmp]} \ ?_{[+exh]} \varphi]\)

- In the case of polar interrogatives all readings coincide, as desired.
- The account so far is quite close to other existing accounts.
- It just breaks the derivation of the three traditional readings down into two steps:
  - First we compute the set of propositions that (exhaustively) specify the property expressed by the prejacent of the interrogative clause, world-independently;
  - Then we compute the set of (complete) suitable answers to the embedded clause in each particular world.

- This decomposition makes it possible to capture FA sensitivity in a simple and uniform way.

3 Capturing false answer sensitivity

3.1 A more stringent notion of suitable answers

- We saw that the set of suitable answers to \(\psi\) in \(w\), as formalized above, is downward closed.
- However, upon closer examination, this is not appropriate.
- To see this, consider George’s scenario again:
  - In \(w\), Italian newspapers are sold at Newstopia and Cellulose City, but not at Paperworld.
  - Let \(\psi\) be the clause \([?_{[-exh]} \text{ where can one buy an Italian newspaper}]\)
  - Now consider:
    \begin{align*}
    (6) \quad &\text{One can buy an Italian newspaper at Newstopia.} \\
    (7) \quad &\text{One can buy an Italian newspaper at Newstopia and at Paperworld.}
    \end{align*}
  - (6) should count as a suitable answer, but (7) shouldn’t.
  - However, if (6) is indeed classified as a suitable answer and the set of suitable answers is downward closed, then (7) inevitably has to be classified as a suitable answer as well.

- So, while downward closure is desirable at the level of resolving propositions, it is not desirable at the level of suitable answers in a particular world.
- Capturing FA sensitivity amounts to preventing downward closure in exactly the right circumstances.
- To do so, we strengthen the notion of (complete) suitable answers as follows:
A proposition is a **complete** suitable answer to a clause $\psi$ in a world $w$ iff

1. it confirms at least one (every) alternative in $\text{ALT}(\psi)$ that is true in $w$, and
2. it does not confirm any alternative in $\text{ALT}(\psi)$ that is false in $w$.

- Formally:
  
  $A_{[-\text{cmp}]} := \lambda S.T.\lambda w.\lambda p. \left( \exists q \in \text{ALT}(S). (q(w) \land p \subseteq q) \land \neg \exists q \in \text{ALT}(S). (\neg q(w) \land p \subseteq q) \right)$

  $A_{[+\text{cmp}]} := \lambda S.T.\lambda w.\lambda p. \left( \forall q \in \text{ALT}(S). (q(w) \rightarrow p \subseteq q) \land \neg \exists q \in \text{ALT}(S). (\neg q(w) \land p \subseteq q) \right)$

- Indeed, under this more stringent notion, there are cases in which the set of suitable answers is **no longer downward closed**.

- In particular, in the above scenario, (6) counts as a suitable answer, but (7) doesn’t.

### 3.2 Interrogative embedding verbs

- So far, we focused on the semantics of interrogative complement clauses.

- Now we turn to the verbs that take such clauses as their argument.

- Such verbs are generally taken to fall into two broad classes: (Groenendijk and Stokhof, 1984)
  - Extensional verbs, e.g., *know, remember*
  - Intensional verbs, e.g., *wonder, be certain*

- The class of intensional verbs can be further divided into: (cf., Lahiri, 2002)
  - Rogative intensional verbs, e.g., *wonder, investigate*
  - Responsive intensional verbs, e.g., *be certain, agree*

- In a diagram:

```
\[\text{extensional} \quad \begin{array}{c}
    \text{know, remember} \\
    \text{intensional} \\
    \text{rogative} \\
    \text{wonder, investigate} \\
    \text{responsive} \\
    \text{be certain, agree} \\
\end{array} \]
```

- Extensional versus intensional
- **Extensional** verbs express a relation to the **extension** of the complement clause $\psi$ in the world of evaluation $w$, which traditionally is the **true complete answer** to $\psi$ in $w$.\(^2\)

\((8)\) John knows who called.
$\rightarrow$ true iff John knows the true complete answer to *who called*

- **Intensional** verbs express a relation to the **intension** of $\psi$, which does not only determine the true complete answer to $\psi$ in $w$, but also all **possible complete answers**.

\((9)\) John is certain who called.
$\rightarrow$ true iff John is certain about a possible complete answer to *who called*,
not necessarily the true one

\((10)\) John wonders who called.
$\rightarrow$ true iff John wants to know, and doesn’t know yet, which of the possible complete answer to *who called* is true

**Responsive versus rogative**

- **Responsive** verbs take both **declarative** and **interrogative** complements.

\((11)\) a. John is certain who called.
b. John is certain that Bill called.

- **Rogative** verbs only take interrogative complements.

\((12)\) a. John wonders who called.
b. *John wonders that Bill called.

**Extensional versus intensional verbs in our setting**

- For us, an interrogative complement clause $\psi$ is of type $\langle s, T \rangle$.
- That is, $\psi$ expresses a function from worlds to sets of propositions.
- This function can be thought of as the **intension** of $\psi$.
- The **extension** of $\psi$ in a world $w$ is obtained by applying the intension of $\psi$ to $w$.
- So the extension of $\psi$ in $w$ is a set of propositions: the set of suitable answers to $\psi$ in $w$.
- So the traditional distinction between extensional and intensional verbs can be preserved.
- Only, the intension and the extension of the complement $\psi$ are different in nature here:
  * they do not capture what the **true** complete answer to $\psi$ is in every world
  * but rather what the **suitable** answers to $\psi$ are in every world

---

\(^2\) Extensional verbs are also sometimes referred to as **veridical-responsive** verbs, or verbs that are **veridical with respect to their interrogative complements** (see, e.g., Lahiri, 2002; Égré and Spector, 2014). This terminology emphasizes that extensional verbs are traditionally taken to express a relation to the **true** complete answer to their complement. In our setting, extensional verbs will still be treated as expressing a relation to the **extension** of their complement, but this extension will no longer be the true complete answer. Thus, our terminological choice here is not arbitrary.
This approach is:
* **More flexible**: it allows us to deal with variance in exhaustive strength
* **More fine-grained**: it allows us to capture false answer sensitivity

- We now give a concrete lexical entry for one verb from each class.

### 3.2.1 Extensional verbs: the case of know

- For every individual $x$ and every world $w$, let $\sigma_x^w$ denote the information state of $x$ in $w$, i.e., the set of worlds compatible with the information available to $x$ in $w$.

- Lexical entry for know:

  \[
  [\text{know}] := \lambda f_{(s,T)}, \lambda x, \lambda p. \forall w : \sigma_x^w \in f(w)
  \]

  In words, know takes:
  - a function $f$ from worlds to sets of suitable answers to the complement clause
  - an individual $x$

  and yields:
  - a set of propositions $p$, such that
  - for every world $w$ in $p$
  - the information state of $x$ in $w$ coincides with a suitable answer in $f(w)$

- Comparison with the standard account of know:

  - On the standard account, $f(w)$ is the complete answer in $w$.
  - It is then required that $\sigma_x^w$ be included in $f(w)$.
  - That is, $x$ should have enough information to establish $f(w)$.
  - For us, $f(w)$ is the set of suitable answers in $w$.
  - We require that $\sigma_x^w$ is an element of $f(w)$.
  - This way we capture that $x$ should have enough information, but not too much.
  - This is crucial to account for false answer sensitivity.

- Example: George’s scenario

  - Recall:

<table>
<thead>
<tr>
<th></th>
<th>Newstopia</th>
<th>Cellulose City</th>
<th>Paperworld</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts in the actual world</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>What Janna believes</td>
<td>Y</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>What Rupert believes</td>
<td>Y</td>
<td>-</td>
<td>Y</td>
</tr>
</tbody>
</table>
(13) Janna knows where one can buy an Italian newspaper. ⇒ true
(14) Rupert knows where one can buy an Italian newspaper. ⇒ false

– Assume that the embedded clause involves $?_{[-\text{exh}]}$ and $A_{[-\text{cmp}]}$, resulting in a mention-some reading.
– (13) is correctly predicted to be true because Janna’s information state coincides with a suitable answer to the embedded clause in the actual world.
– (14) is correctly predicted to be false because Rupert’s information state does not coincide with a suitable answer to the embedded clause in the actual world.

Thus, false answer sensitivity of know is captured.

3.2.2 Responsive intensional verbs: the case of be certain

• For every individual $x$ and every world $w$, let $c^w_x$ denote the set of worlds that are compatible with what $x$ is certain about in $w$.

• Lexical entry for be certain:

\[
[\text{be certain}] := \lambda f_{(s,T)\alpha x} . \lambda p . \exists v : \forall w \in p : c^w_x \in f(v)
\]

In words, be certain takes:

– a function $f$ from worlds to sets of suitable answers to the complement clause
– an individual $x$

and yields:

– a set of propositions $p$, such that

– for some world $v$

– it holds for every $w \in p$ that

– $c^w_x$ coincides with a suitable answer in $f(v)$ (rather than $f(w)$)

• Comparison with know

– Crucially, in the case of responsive intensional verbs we are allowed to consider other worlds than the world of evaluation.

– As a consequence, responsive intensional verbs exhibit no false answer sensitivity.

• Example: a variant of George’s scenario

– The same scenario, but now involving certainty rather than belief:

<table>
<thead>
<tr>
<th>Where can one buy an Italian newspaper?</th>
<th>Newstopia</th>
<th>Cellulose City</th>
<th>Paperworld</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facts in the actual world</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>What Janna is certain about</td>
<td>Y</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>What Rupert is certain about</td>
<td>Y</td>
<td>-</td>
<td>Y</td>
</tr>
</tbody>
</table>
(15) Janna is certain where one can buy an Italian newspaper. \(\Rightarrow\) true
(16) Rupert is certain where one can buy an Italian newspaper. \(\Rightarrow\) true

– Now both sentences are true, even though Rupert is certain that Italian newspapers are sold at Paperworld, which is actually false.
– Assume again that the embedded clause involves \(?\_{exh}\) and \(A\_{cmp}\), resulting in a mention-some reading.
– (15) is correctly predicted to be true because Janna’s ‘certainty state’ coincides with a suitable answer to the embedded clause in the actual world.
– (16) is correctly predicted to be true because Rupert’s ‘certainty state’ coincides with a suitable answer to the embedded clause in a non-actual world, namely one where Italian newspapers are sold at Newstopia and at Paperworld.
– Thus, it is correctly predicted that responsive intensional verbs do not exhibit false answer sensitivity.

### 3.2.3 Rogative intensional verbs: the case of wonder

- We adapt the account of wonder given in Ciardelli and Roelofsen (2014b) to our current setting, where the analysis of interrogative complements is more fine-grained.
- To formally capture what it means to wonder about something, it is necessary to not only model the information state of an individual, but also her inquisitive state.
- We will denote the inquisitive state of an individual \(x\) in a world \(w\) as \(\Sigma^w_x\).
- \(\Sigma^w_x\) can be modeled as an issue over \(\sigma^w_x\), in the sense of inquisitive semantics.
- That is:
  - \(\Sigma^w_x\) is a non-empty, downward closed set of propositions/information states.
  - \(\Sigma^w_x\) forms a cover of \(\sigma^w_x\): \(\bigcup \Sigma^w_x = \sigma^w_x\)
- We think of the elements of \(\Sigma^w_x\) as information states that \(x\) wants to reach.
- Informally, then, \(x\) wonders about \(\psi\) just in case:
  - In \(x\)’s current information state, no suitable answer to \(\psi\) is established yet;
  - In every information state that \(x\) wants to reach, a suitable answer to \(\psi\) is established.
- Lexical entry for wonder:
  \([\text{wonder}] := \lambda f \langle s,T \rangle. \lambda x. \lambda p. \forall w \in p : (\sigma^w_x \notin \bigcup_{v \in W} f(v) \land \Sigma^w_x \subseteq \bigcup_{v \in W} f(v))\)
In words, wonder takes:
  - a function \(f\) from worlds to sets of suitable answers to the complement clause
  - an individual \(x\)
and yields:
– a set of propositions $p$, such that
– for every world $w$ in $p$:
  (i) $\sigma^w_x$ does not coincide with any suitable answer in any world
  (ii) every state in $\Sigma^w_x$ does coincide with a suitable answer in some world

• Comparison with know and be certain
– Just like in the case of responsive intensional verbs, we consider suitable answers to the
  embedded clause in other worlds than the world of evaluation.
– As a consequence, rogative intensional verbs, just like responsive intensional verbs, do
  not exhibit false answer sensitivity.

4 Declarative complements

4.1 Basic syntactic and semantic assumptions
• We want to treat declarative and interrogative complements in a uniform way.
• Thus, in both cases, we assume the following structure:

```
  matrix clause
    subject
        embedding verb
            embedded clause
                $A_{[+/-\text{cmp}]}$
        declarative/interrogative clause
```

• In particular, we assume that the $A$ operator is present in both cases.
• The challenge is to define it in such a way that it gives the right results for both kinds of
  complements. This will require a refinement of the current definition.
• Zooming in on the clause that $A$ takes as its input, we assume that:
  – Interrogative clauses are headed by the $?^n$ operator, either $+$ or $-$ exhaustive;
  – Declarative clauses are headed by a non-inquisitive projection operator, denoted $!$.

\[
\begin{align*}
\text{declarative} & : ! \quad \langle T, T \rangle \\
\text{polar interrogative} & : ?^0 \quad \langle T, T \rangle \\
\text{n-place interrogative} & : ?^m \quad \langle \langle e^n, T \rangle, T \rangle
\end{align*}
\]
Sentential meanings, i.e., semantic objects of type $T$, can be thought of in inquisitive semantics as inhabiting a two-dimensional space:

![Inquisitive Informative Diagram]

- **Projection operators** are operators that obliterate one dimension of meaning, while leaving the other intact.

- In particular, a **non-inquisitive** projection operator is one that obliterates inquisitiveness while leaving informative content in tact.

- Thus, if $!$ is to be a non-inquisitive projection operator, then for any expression $\varphi$ of type $T$:
  - $!\varphi$ should be **non-inquisitive**, and
  - $!\varphi$ should have the **same informative content** as $\varphi$ itself.

- It turns out that there is exactly one way to define $!$ that achieves this result:
  \[
  ! := \lambda S. \lambda p. \forall w \in p : \{w\} \in S
  \]

- In terms of alternatives, $[!\varphi]$ always contains a **single alternative**, which is the **union** of all the alternatives for $\varphi$.

- For instance:

  ![Ann or Bill called](image)
  ![Ann or Bill called](image)

- The fact that the semantic value of a declarative always contains a **single alternative** crucially distinguishes it from interrogatives.

---

3For a proof of this claim, see Roelofsen (2013).
4.2 Factivity presuppositions

- Verbs like *know* trigger a **factivity presupposition** when taking a declarative complement:

  (17) John knows that Mary called.

  \[ \sim \text{presupposition: Mary called.} \]

- Égré and Spector’s (2014) generalization

  The class of verbs that give rise to **factivity** presuppositions when taking a declarative complement is precisely the class of **extensional** verbs.\(^4\)

- **Our strategy to derive factivity presuppositions**

  - The *A* operator, when applied to a clause \( \psi \), yields a function \( f \) that maps every world to the set of suitable answers to \( \psi \) in that world.
  - We have so far assumed that \( f \) is defined for every world \( w \).
  - But it makes sense to assume that it is sometimes **undefined**.
  - In particular, if \( w \) is not included in any alternative for \( \psi \).
  - In the case of a declarative, this means that \( f(w) \) is undefined if \( \psi \) is false in \( w \).
  - Now, *know* takes \( f \) as its input, plus an individual \( x \), and yields a set of propositions \( S \).
  - To determine whether \( p \in S \), we look at every \( w \in p \) and check whether \( \sigma^w_x \in f(w) \).
  - But what if \( f(w) \) is undefined?
  - Then we cannot determine whether \( p \) is in \( S \) either.
  - This is what gives rise to a factivity presupposition.

- More formally, we add a **definedness restriction** to the definition of the *A* operator:

  (the underlined part is to be read as *this term is only defined if...*)

\[
A_{[+\text{cmp}]} := \lambda S. \lambda w. \lambda p. \exists q \in \text{ALT}(S) : q(w) . \left( \forall q \in \text{ALT}(S) : (q(w) \rightarrow p \subseteq q) \land \forall q \in \text{ALT}(S) : (-q(w) \rightarrow p \not\subseteq q) \right)
\]

\[
A_{[-\text{cmp}]} := \lambda S. \lambda w. \lambda p. \exists q \in \text{ALT}(S) : q(w) . \left( \exists q \in \text{ALT}(S) : (q(w) \land p \subseteq q) \land \forall q \in \text{ALT}(S) : (-q(w) \rightarrow p \not\subseteq q) \right)
\]

- If *A* applies to a **declarative**, then \( \text{ALT}(S) \) contains a **single alternative**.

- In this case, \( A_{[+\text{cmp}]} \) and \( A_{[-\text{cmp}]} \) coincide and simply amount to:

\[
A_{[+/\text{-cmp}]} := \lambda S. \lambda w. \lambda p. \exists q \in \text{ALT}(S) : q(w) . \exists q \in \text{ALT}(S) : (q(w) \land p \subseteq q)
\]

---

\(^4\)The controversial cases are **communication verbs** such as *tell, predict*, and *announce*. Many authors have assumed that such verbs are **extensional** but to **not** give rise to **factivity presuppositions** (e.g., Karttunen, 1977; Groenendijk and Stokhof, 1984; Berman, 1991; Higginbotham, 1996; Lahiri, 2002). Égré and Spector argue, building on Tsotzis (1993) and Schlenker (2006), that such verbs exhibit a systematic ambiguity: under one interpretation they are extensional and give rise to factivity presuppositions, while on the other interpretation they are not extensional and do not give rise to factivity presuppositions either. Theiler (2014) argues that this ambiguity is a particular instance of a more general ambiguity between **literal** and **deductive** readings of verbs that take a clausal complement.
• We also build a definedness restriction into the semantics of **extensional** verbs like *know*:

\[
[\text{know}] := \lambda f_{(s,T)}. \lambda x. \lambda p. \forall w \in p : f(w) \text{ is defined.} \forall w \in p : \sigma^w_x \in f(w)
\]

• For any expression \( \varphi \) of type \( T \), we say that the **presupposition** of \( \varphi \) is the set of all worlds \( w \) such that \( [[\varphi]](\{w\}) \) is defined.

• What does this mean for example (17)?
  - (17) expresses a partial function from propositions to truth values
  - \( [(17)](p) \) is undefined iff in some world in \( p \), Mary didn’t call
  - So the presupposition of (17) consists of all worlds where Mary called

• The definedness restrictions that we implemented account for the factivity presuppositions triggered by extensional verbs like *know* with a declarative complement.

• But what other predictions does the account make?
  - What happens when an extensional verb takes an **interrogative complement**?
    \( \rightarrow \) Do we predict some kind of **factivity presupposition** in this case as well?
  - What about **intensional** verbs?
    \( \rightarrow \) Why do some of these **not take declarative complements** at all?
    \( \rightarrow \) Why are those that do take declarative complements **not factive**?
  - What about the **false answer sensitivity** effects that we predicted for interrogatives?
    \( \rightarrow \) Are such effects also predicted for **declaratives**?

### 4.3 Declarative versus interrogative complements

• The \( A \) operator applies uniformly both to declarative and to interrogative clauses.

• However, declarative and interrogative clauses have **different semantic properties**.

• The \( A \) operator is sensitive to this, and yields **different results** in the two cases.

• In examining these differences, we will find the following:

<table>
<thead>
<tr>
<th></th>
<th>false answer sensitivity</th>
<th>factivity presupposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>interrogative</td>
<td>yes</td>
<td>trivial</td>
</tr>
<tr>
<td>declarative</td>
<td>no</td>
<td>non-trivial</td>
</tr>
</tbody>
</table>

#### 4.3.1 False answer sensitivity

• Suppose that only **Alice** called.

• Further suppose John wrongly believes that **Alice and Bob** called.

• Then (18) is judged false, whereas (19) gives rise to a presupposition failure.
(18) John knows who called. ⇒ false
(19) John knows that Alice and Bob called. ⇒ presupposition failure

- With interrogative complements, false answers have **truth-conditional effects**.  
  → These are handled by the FA sensitive semantics of the $A$ operator and the 'exact match semantics' of the embedding verb.

- With declarative complements, false answers have **non-truth-conditional effects**.  
  → These are handled by the definedness restrictions of the $A$ operator and the verb.

Interrogative complements

- The semantic value of an interrogative clause is a set of propositions $S$, typically containing multiple alternatives.
- The $A$ operator extracts from this set only those propositions that do not confirm any $q \in \text{alt}(S)$ that is false in $w$.
- Since $S$ typically contains multiple alternatives, it also typically contains alternatives that are false in $w$.
- Hence, if $A$ applies to interrogatives, the **FA condition** typically applies **non-vacuously**.

Declarative complements

- The semantic value of an declarative clause is a set of propositions $S$, containing a single alternative. Let $q$ be this alternative.
- Given a world $w$, we can distinguish two cases: $w \in q$ or $w \notin q$.
- **Case 1**: $w \in q$,
  - In this case there exists no alternative in $S$ that does not contain $w$.
  - Recall the FA sensitivity condition in the definition of the $A$ operator:
    $$\neg \exists q \in \text{alt}(S) : (\neg q(w) \land p \subseteq q)$$
  - If $w \in q$, this condition is **trivially satisfied** for all propositions $p$.
  - This means that all propositions from $S$ are included in $A(S)(w)$.
- **Case 2**: $w \notin q$
  - In this case, $A(S)(w)$ is not defined.
  - Hence, the FA condition does not come into play.
- Thus, if $A$ applies to a declarative, the **FA condition** applies **vacuously**, or not at all.
4.3.2 Factivity presuppositions

Interrogative complements

- The ? operator ensures that an interrogative clause \( \psi \) is never informative.
- This means that the alternatives in \( \text{ALT}(\langle \psi \rangle) \) always cover the entire semantic space.
- Thus, for any world \( w \), there exists an alternative \( q \in \text{ALT}(\langle \psi \rangle) \) such that \( q(w) \).
- Hence, \( A(\langle \psi \rangle)(w) \) is defined for all \( w \).
- This means that the factivity presupposition is trivially satisfied.

Declarative complements

- A declarative clause is typically informative.
- This means that there exist worlds \( w \) which are contained in none of the alternatives in \( \text{ALT}(\langle \psi \rangle) \).
- For these worlds, \( A(\langle \psi \rangle)(w) \) is undefined.
- This means that, with declarative complements, the factivity presupposition is non-trivial.

4.4 Declarative complements and intensional verbs

- What happens if an intensional verb takes a declarative complement?
- Rogative intensional verbs like \textit{wonder} do not license declarative complements at all.
  \begin{equation}
  \text{(20)} \quad \ast \text{John wonders that Mary called.}
  \end{equation}
- Responsive intensional verbs like \textit{be certain} license declarative complements but do not trigger factivity presuppositions.
  \begin{equation}
  \text{(21)} \quad \text{John is certain that Mary called.}
  \end{equation}
  \( \leadsto \) does not presuppose that Mary called.
- Is this predicted?

4.4.1 Rogative intensional verbs: the case of \textit{wonder}

- Recall our lexical entry for \textit{wonder}, adapted from Ciardelli and Roelofsen (2014b):
  \[ [\text{wonder}] := \lambda f_{(s,T)} \lambda x. \lambda p. \forall w \in p : (\sigma_w^x \notin \bigcup_{v \in W} f(v) \land \Sigma^w_x \subseteq \bigcup_{v \in W} f(v)) \]
- Ciardelli and Roelofsen (2014b) show that (a simplified version of) this entry accounts for the fact that \textit{wonder} does not license declarative complements.
- To see this, recall that the \textbf{inquisitive state} of \( x \) in \( w \), \( \Sigma^w_x \), is a set of information states which together form a \textbf{cover} of \( \sigma^w_x \):
  \[ \bigcup \Sigma^w_x = \sigma^w_x \]
Moreover, recall that the semantic value of a declarative clause is a set of propositions $S$ containing a single alternative $q$.

Now, if we apply $A$ to $S$ we get a function $f$ from worlds to sets of propositions, such that:

- If $w \in q$, then $f(w)$.
- If $w \notin q$, then $f(w)$ is undefined.

Consider the following two cases:

- **Case 1:** $q = \emptyset$, i.e., the declarative clause is a contradiction.
  - Then $f(w)$ is undefined for all $w$.
  - This means that $\bigcup_{v \in W} f(v) = \emptyset$.
  - But then the second conjunct in the lexical entry for wonder can never be satisfied.
  - Thus, applying wonder to the declarative complement yields a contradiction.

- **Case 1:** $q \neq \emptyset$, i.e., the declarative clause is not a contradiction.
  - Then for all $w \in q$, $f(w)$ is defined and amounts to $\wp(q)$.
  - For all $w \notin q$, $f(w)$ is undefined.
  - This means that $\bigcup_{v \in W} f(v) = \wp(q)$.
  - Now, since $\wp(q)$ is a powerset, it is **closed under unions**: for any $S \subseteq \wp(q)$, $\bigcup S \in \wp(q)$
  - Now, suppose that the second conjunct in the lexical entry for wonder is satisfied:
    \[
    \Sigma^w_x \subseteq \bigcup_{v \in W} f(v) \quad (= \wp(q))
    \]
    - Then, since $\wp(q)$ is closed under unions, it must also be the case that:
      \[
      \bigcup \Sigma^w_x \in \bigcup_{v \in W} f(v)
      \]
    - But then, since $\bigcup \Sigma^w_x = \sigma^w_x$, we have that:
      \[
      \sigma^w_x \in \bigcup_{v \in W} f(v)
      \]
    - This means that the first conjunct in the lexical entry for wonder is not satisfied.
    - So the conjunction as a whole cannot hold.

Thus, with a declarative complement, wonder always yields a contradiction.

This explains why wonder does not license declarative complements.

A similar explanation can be given for (at least some) other rogative verbs (e.g., investigate, depend on).
4.4.2 Responsive intensional verbs: the case of be certain

- Recall our non-presuppositional lexical entry for be certain:
  \[
  \text{[be certain]} := \lambda f_{(s,T)}.\lambda x.\lambda p. \exists v : \forall w \in p : c^w_x \in f(v)
  \]

- Compare this with our (obsolete) non-presuppositional entry for know
  \[
  \text{[know]} := \lambda f_{(s,T)}.\lambda x.\lambda p. \forall w \in p : \sigma^w_x \in f(w)
  \]

- Crucial difference
  - be certain looks at the set of suitable answers in some world \(v\), which can be chosen independently of \(p\)
  - know looks at the set of suitable answers in every world \(w \in p\).

- For know, we added the presupposition that for every \(w \in p\), the set of suitable answers is well-defined.

- Similarly, for be certain it makes sense to add the presupposition that in \(v\), the set of suitable answers is well-defined.

- Presuppositional lexical entry for be certain
  \[
  \text{[be certain]} := \lambda f_{(s,T)}.\lambda x.\lambda p. \exists v : f(v) \text{ is defined} \exists v : \forall w \in p : c^w_x \in f(v)
  \]

- However, since \(v\) can be picked freely, this presupposition is much less demanding.

- The presupposition is just that in some world, the complement has a well-defined set of suitable answers.

- For a declarative complement, this means that there should be some world in which the declarative is true.

- Thus, presupposition failure only arises if the complement is contradictory.

Taken together, these considerations explain Égré and Spector’s generalization: the class of verbs that give rise to factivity presuppositions coincides precisely with the class of extensional verbs.

5 Conclusion

- We outlined an account of interrogative and declarative complements and the different kinds of verbs that take such complements.

- The account uniformly captures the false answer sensitivity exhibited by extensional verbs with interrogative complements, across different levels of exhaustive strength.

- It also predicts that false answer sensitivity does not arise with intensional verbs and with declarative complements.

- On the other hand, it captures the factivity presuppositions exhibited by extensional verbs with declarative complements.
• It further predicts that factivity presuppositions do not arise with intensional verbs and with interrogative complements.

• Thus, an intrinsic connection emerges between:
  – Extensionality
  – False answer sensitivity (in the case of interrogative complements)
  – Factivity presuppositions (in the case of declarative complements)

• False answer sensitivity and factivity presuppositions are derived by means of a single operator, which is taken to be present both in declarative and in interrogative complements.

• For the class of intensional verbs, the account predicts that FA sensitivity and factivity presuppositions do not arise.

• Moreover, it is predicted that certain intensional verbs, e.g. *wonder*, do not license declarative complements at all.

References


