A unified account of the inferences of disjunction under modals

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Disjunction Days, Berlin, 2016

Outline

Introduction

The inferences of disjunction and modals

- Disjunction in the scope of modals invariably gives rise to the inferences that its disjuncts are possible.

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a. $\rightsquigarrow \Diamond A \wedge \Diamond B$

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(1) $\textsc{mod}(A \vee B)$

a. $\rightsquigarrow \Diamond A \wedge \Diamond B$

- This is true regardless of the force of $\textsc{mod}$

Introduction

The inferences of disjunction and modals

(2) It's certain that it will rain or snow

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(3) It's likely that it will rain or snow

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(2) It's certain that it will rain or snow

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- All suggest that rain is possible and snow is possible.

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The inferences of disjunction and modals

(2) It's certain that it will rain or snow

(3) It's likely that it will rain or snow

(4) It's possible that it will rain or snow

- All suggest that rain is possible and snow is possible.
- All sound odd if e.g. snow is impossible.

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The inferences of disjunction and modals

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- The distributive inferences of *certain* and *likely* are derived straightforwardly by most accounts.
- The free choice inferences of *possible* are more problematic.

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The inferences of disjunction and modals

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  (Shimoyama and Kratzer 2002, Fox 2007, Alonso-Ovalle 2006, Klinedinst 2007, Chemla 2010 a.o.)

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- In particular for their disappearance in DE contexts.

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$\not\rightsquigarrow$ It's not [possible rain and possible snow]

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- In particular for their disappearance in DE contexts.

(5) It's not possible that it will rain or snow
$\not\rightsquigarrow$ It's not [possible rain and possible snow]

- While both treated as implicatures, they are nonetheless treated differently.

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The inferences of disjunction and modals

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The inferences of disjunction and modals

- We show that an independently motivated degree-based semantics for modals can predict all these inferences via the same mechanism.
- $\textsc{mod}(A \vee B) \wedge \neg\textsc{mod}(A) \wedge \neg\textsc{mod}(B)$
$\rightsquigarrow \Diamond(A) \wedge \Diamond(B)$

Introduction

The inferences of disjunction and modals

- We focus on epistemic modal adjectives.
- We sketch how the account can be extended to modals of other syntactic categories and different modal flavors.

The plan

- The problem of free choice inferences

The plan

- The problem of free choice inferences
- Overview of the account

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- Overview of the account
- Background on gradable adjectives

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- The account in more detail

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- Overview of the account
- Background on gradable adjectives
- The account in more detail
- Extensions and other cases
- Skepticism about gradability and probabilistic implementation

Outline

Distributive inferences

(6) It is certain that it will rain or snow
$\rightsquigarrow$ it's possible it will rain and it's possible it will snow

Distributive inferences

(6) It is certain that it will rain or snow
$\rightsquigarrow$ it's possible it will rain and it's possible it will snow

(7) It is likely that it will rain or snow
$\rightsquigarrow$ it's possible it will rain and it's possible it will snow

Scalar implicatures

- Assume an exhaustification-based analysis of scalar implicatures.

Scalar implicatures

- Define EXH in a standard way

(8) $[\![\text{exh}]\!](p)(w) =$
$p(w) \wedge \forall q \in Excl(p, Alt(p))[\neg q(w)]$

Scalar implicatures

- Define EXH in a standard way

(8) $[\![\text{exh}]\!](p)(w) =$
$p(w) \wedge \forall q \in \mathit{Excl}(p, \mathit{Alt}(p))[\neg q(w)]$

(9) $\mathit{Excl}(p, Q) =$
$\{q \in Q : p \not\subseteq q \wedge \neg\exists r[r \in Q \wedge (p \wedge \neg q) \subseteq r\}$

Alternatives

- Assume that a disjunctive sentence like (10) has the alternatives in (11). (Sauerland 2004, Katzir 2007, Chemla 2010 a.o.)

(10) $A \vee B$

(11) $\{(A \vee B), A, B, (A \wedge B)\}$

Example

- Example: EXH applied to a simple disjunctive sentence.

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(12) $\text{EXH}(A \vee B)$

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(12) $\text{EXH}(A \vee B)$

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- In this case, EXH just negates the conjunctive alternative.

(14) $\text{EXH}(A \vee B) = (A \vee B) \wedge \neg(A \wedge B)$

Distributive inferences

- Now, back to distributive inferences. Consider again:

(15) It is certain that it will rain or snow

Distributive inferences

- Now, back to distributive inferences. Consider again:

(15) It is certain that it will rain or snow

(16) $\approx \Box(r \vee s)$

Distributive inferences

(17) $\textsc{exh}(\Box(r \vee s))$

Distributive inferences

(17) $\textsc{exh}(\Box(r \vee s))$

(18) $\mathsf{Alt} = \{\ldots \Box r, \Box s, \Box(r \wedge s)\}$

Distributive inferences

(17) $\text{EXH}(\Box(r \vee s))$

(18) $\text{Alt} = \{\ldots \Box r, \Box s, \Box(r \wedge s)\}$

(19) $\Box(r \vee s) \wedge \neg\Box r \wedge \neg\Box s =$

Distributive inferences

(17) $\text{EXH}(\Box(r \vee s))$

(18) $\text{Alt} = \{\ldots \Box r, \Box s, \Box(r \wedge s)\}$

(19) $\Box(r \vee s) \wedge \neg\Box r \wedge \neg\Box s =$
$\Box(r \vee s) \wedge \Diamond r \wedge \Diamond s$

Outline

Free choice inferences

(20) It is possible that it will rain or snow

(21) $\approx \Diamond(r \vee s)$

Free choice inferences

(22) $\textsc{exh}(\Diamond(r \vee s))$

Free choice inferences

(22) $\textsc{exh}(\Diamond(r \vee s))$

(23) $\text{Alt} = \{\ldots \Diamond r, \Diamond s, \Diamond(r \wedge s)\}$

Free choice inferences

(22) $\textsc{exh}(\Diamond(r \vee s))$

(23) $\mathrm{Alt} = \{\ldots \Diamond r, \Diamond s, \Diamond(r \wedge s)\}$

(24) $\Diamond(r \vee s) \wedge \neg\Diamond r \wedge \neg\Diamond s = \bot$

Free choice inferences

(22) $\textsc{exh}(\Diamond(r \vee s))$

(23) $\text{Alt} = \{\ldots \Diamond r, \Diamond s, \Diamond(r \wedge s)\}$

(24) $\Diamond(r \vee s) \wedge \neg\Diamond r \wedge \neg\Diamond s = \bot$

(25) $\Diamond(r \vee s) \wedge \neg\Diamond(r \wedge s)$

Free choice inferences

- We do not get the free choice inference in (27).

(26) It's possible that it will rain or snow

(27) $\rightsquigarrow$ It's possible that it will rain and it's possible that it will snow

Free choice inferences

- A variety of responses from the scalar implicature approach.

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Free choice inferences

- A variety of responses from the scalar implicature approach.
  - complicating the theory of scalar implicatures (e.g., recursive exhaustification, similarity . . .) Fox 2007, Chemla 2010 a.o.
  - complicating the theory of modality (modals as plurals over worlds) Klinedinst 2006 a.o.
- Our account: treating modals as degree expressions and exhaustifying above the modal itself, but below a positive morpheme

Outline

The idea in a nutshell

- Epistemic modal adjectives are gradable expressions relating propositions to degrees.

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- The scale they use is that of probability. (Yalcin 2010, Lassiter 2011, a.o.)

The idea in a nutshell

- They combine with a covert morpheme POS, which existentially quantifies over the degree variable.

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- They combine with a covert morpheme POS, which existentially quantifies over the degree variable.
- Exhaustification can occur above or below POS.

The idea in a nutshell

- Consider a simple sentence like (28).

(28) It is likely that it will rain

The idea in a nutshell

- Consider a simple sentence like (28).

(28) It is likely that it will rain

(29) It is POS [likely that it will rain]

The idea in a nutshell

- Consider a simple sentence like (28).

(28) It is likely that it will rain

(29) It is POS [likely that it will rain]

- Truth conditions of (28): the probability of rain exceeds the contextual standard for likely.

The idea in a nutshell

- Similarly for *certain* and *possible*.

(30) It is certain that it will rain

The idea in a nutshell

- Similarly for *certain* and *possible*.

(30) It is certain that it will rain

(31) It is POS [certain that it will rain]

The idea in a nutshell

- Similarly for *certain* and *possible*.

(30) It is certain that it will rain

(31) It is POS [certain that it will rain]

- Truth conditions of (30): the probability of rain is 1.

- Similarly for *certain* and *possible*.

(32) It is possible that it will rain

- Similarly for *certain* and *possible*.

(32) It is possible that it will rain

(33) It is POS [possible that it will rain]

- Similarly for *certain* and *possible*.

(32) It is possible that it will rain

(33) It is POS [possible that it will rain]

- Truth conditions of (32): the probability of rain is greater than 0.

The idea in a nutshell

- When we have disjunction as in (34)

(34) It is likely that it will rain or snow

- The probability of rain or snow exceeds the contextual standard for likely.

The probability of a disjunction

- Fact about probability: the probability of a disjunction upper-bounds the probabilities of the disjuncts.

(35) $Pr(A), Pr(B) \leq Pr(A \vee B)$

The idea in a nutshell

- Consider now embedding an EXH below POS:

(36) ⟦POS[EXH[...likely that it will rain or snow]]⟧

The idea in a nutshell

- Consider now embedding an EXH below POS:

(36) $[\![$POS[EXH[...likely that it will rain or snow]]$]\!]$

(37) { ...likely it will rain, likely it will snow ... }

The idea in a nutshell

- The result of exhaustification:

(38) The probability of rain or snow exceeds the contextual standard for likely

The idea in a nutshell

- The result of exhaustification:

(38) The probability of rain or snow exceeds the contextual standard for likely
but the probability of rain does not

The idea in a nutshell

- The result of exhaustification:

(38) The probability of rain or snow exceeds the contextual standard for likely
but the probability of rain does not
and the probability of snow does not either

The idea in a nutshell

- Now: if the probability of rain or snow is greater than d,

The idea in a nutshell

- Now: if the probability of rain or snow is greater than d,
- but the probability of each of rain and snow is at most as high as d,

The idea in a nutshell

- Now: if the probability of rain or snow is greater than d,
- but the probability of each of rain and snow is at most as high as d,
- then each of the two disjunct has to contribute some positive amount of probability to the probability of the disjunction.

The idea in a nutshell

- More abstractly

(39) $Pr(r \vee s) > d$

The idea in a nutshell

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(39) $Pr(r \vee s) > d$

(40) $Pr(r) \leq d$ and $Pr(s) \leq d$

The idea in a nutshell

- More abstractly

(39) $Pr(r \vee s) > d$

(40) $Pr(r) \leq d$ and $Pr(s) \leq d$

- it follows that $Pr(r) > 0$ and $Pr(s) > 0$.

The idea in a nutshell

- But then, given our analysis of *possible*, the distributive inferences immediately follow,

(41) $\rightsquigarrow$ It's possible that it will rain and it's possible that it will snow

The idea in a nutshell

- The reasoning above extends straightforwardly to *certain*.
- Crucially, it also extends to *possible* in the same way, thus extending to free choice inferences.

Outline

Gradable adjectives

- In the background: we adopt a standard treatment of adjectives like *tall*, *full* and *open*.
- These adjectives relate individuals to degrees,
- and combine with a covert POS in the positive form.

Gradable adjectives

- Consider an adjective like *tall*:

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(42) $[\![\text{tall}]\!] = \lambda d \lambda x[\textsc{height}(x) \geq d]$

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(42) $[\![\text{tall}]\!] = \lambda d \lambda x[\textsc{height}(x) \geq d]$

(43) $[\![\textsc{pos}]\!] = \lambda G_{\langle d,et \rangle} \lambda x \exists d[\mathbf{R}(G)(d) \wedge G(d)(x)]$

Gradable adjectives

- Consider an adjective like *tall*:

(42) $[\![\text{tall}]\!] = \lambda d \lambda x[\textsc{height}(x) \geq d]$

(43) $[\![\textsc{pos}]\!] = \lambda G_{\langle d,et\rangle} \lambda x \exists d[\mathbf{R}(G)(d) \wedge G(d)(x)]$

(44) $[\![\text{John is } \textsc{pos} \text{ tall}]\!] =$
$\exists d[d > s_{tall} \wedge \textsc{height}(j) \geq d]$

Gradable adjectives

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(45) $[\![\text{open}]\!] = \lambda d \lambda x[\textsc{openness}(x) \geq d]$

(46) $[\![\textsc{pos}]\!] = \lambda G_{\langle d,et \rangle} \lambda x \exists d[\mathbf{R}(G)(d) \wedge G(d)(x)]$

Gradable adjectives

- Analogous treatment of *open* and *full* but the context dependence of the threshold.

(45) $[\![\text{open}]\!] = \lambda d \lambda x[\textsc{openness}(x) \geq d]$

(46) $[\![\textsc{pos}]\!] = \lambda G_{\langle d,et \rangle} \lambda x \exists d[\mathbf{R}(G)(d) \wedge G(d)(x)]$

(47) $[\![\text{The door is } \textsc{pos} \text{ open}]\!] =$
$\exists d[d > \textsc{min}_{open} \wedge \textsc{openness}(the - door) \geq d]$

Gradable adjectives

- Same for *full*

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(48) $[\![\text{full}]\!] = \lambda d \lambda x[\textsc{fullness}(x) \geq d]$

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Gradable adjectives

- Same for *full*

(48) $[\![\text{full}]\!] = \lambda d \lambda x[\text{FULLNESS}(x) \geq d]$

(49) $[\![\text{POS}]\!] = \lambda G_{\langle d,et \rangle} \lambda x \exists d[\mathbf{R}(G)(d) \wedge G(d)(x)]$

(50) $[\![\text{The glass is POS full}]\!] =$
$\exists d[d = \text{MAX}_{full} \wedge \text{FULLNESS}(the - glass) \geq d]$

Gradable adjectives

- We extend this treatment to modal adjectives like *certain*, *likely* and *possible*.

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- They map propositions to degrees on a probability scale.

Gradable adjectives

- We extend this treatment to modal adjectives like *certain*, *likely* and *possible*.
- They map propositions to degrees on a probability scale.
- They also combine with a POS operator that introduces existential quantification over degrees.

Gradable adjectives

- The relevant entries:

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(51) $[\![\text{certain}]\!] = [\![\text{likely}]\!] = [\![\text{possible}]\!] = \lambda p \lambda d [Pr(p) \geq d]$

Gradable adjectives

- The relevant entries:

(51) $[\![\text{certain}]\!] = [\![\text{likely}]\!] = [\![\text{possible}]\!] = \lambda p \lambda d[Pr(p) \geq d]$

(52) $[\![\text{POS}]\!] = \lambda G_{\langle d,t \rangle} \exists d[\mathbf{R}(G)(d) \wedge G(d)]$

Gradable adjectives

- The relevant entries:

(51) $[\![\text{certain}]\!] = [\![\text{likely}]\!] = [\![\text{possible}]\!] = \lambda p \lambda d[Pr(p) \geq d]$

(52) $[\![\text{POS}]\!] = \lambda G_{\langle d,t\rangle} \exists d[\mathbf{R}(G)(d) \wedge G(d)]$

- Notice: we assume that some element in the semantics of the adjectives will 'instruct' POS to select different points on a scale in different cases. (Similarly to Kennedy 2007.)

Gradable adjectives

- Going back to sentences involving epistemic modals:

Gradable adjectives

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(53) It's certain that it will rain

(54) It's likely that it will rain

(55) It's possible that it will rain

Gradable adjectives

- Our analysis:

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(56) $[\![$it is POS certain that it will rain$]\!] = 1$ iff $\exists d[d = 1 \wedge Pr(r) \geq d]$

Gradable adjectives

- Our analysis:

(56) $[\![$it is POS certain that it will rain$]\!] = 1$ iff $\exists d[d = 1 \wedge Pr(r) \geq d]$

(57) $[\![$it is POS likely that it will rain$]\!] = 1$ iff $\exists d[d > \mathbf{s}_{likely} \wedge Pr(r) \geq d]$

Gradable adjectives

- Our analysis:

(56) $[\![\text{it is } \textsc{pos} \text{ certain that it will rain}]\!] = 1$ iff
$\exists d[d = 1 \wedge Pr(r) \geq d]$

(57) $[\![\text{it is } \textsc{pos} \text{ likely that it will rain}]\!] = 1$ iff
$\exists d[d > \mathbf{s}_{likely} \wedge Pr(r) \geq d]$

(58) $[\![\text{it is } \textsc{pos} \text{ possible that it will rain}]\!] = 1$ iff
$\exists d[d > 0 \wedge Pr(r) \geq d]$

The scope of POS

- POS and more in general the Degree Phrase can take scope over some things but not others (Heim 2000, Kennedy 1997, Beck 2011, Romero 2015 a.o.)
- The relevant syntactic constraints:

The scope of POS

- POS and more in general the Degree Phrase can take scope over some things but not others (Heim 2000, Kennedy 1997, Beck 2011, Romero 2015 a.o.)
- The relevant syntactic constraints:
  - no quantifiers, no negation
  - but modals okay
  - arguments for movement (e.g., ACD . . .)

The scope of POS

- POS and more in general the Degree Phrase can take scope over some things but not others (Heim 2000, Kennedy 1997, Beck 2011, Romero 2015 a.o.)
- The relevant syntactic constraints:
  - no quantifiers, no negation
  - but modals okay
  - arguments for movement (e.g., ACD . . .)
- These constraints are not very well understood, but generally adopted. (And we adopt them too.)
- But: we assume that POS can take scope over EXH.

Outline

Back to disjunction under epistemic modals

- Put disjunction back in the picture:

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- Put disjunction back in the picture:

(59) It's certain that it will rain or snow

(60) It's likely that it will rain or snow

(61) It's possible that it will rain or snow

The account

- Take a disjunction under *likely*:

(62) It is likely that it will rain or snow

(63) ⟦It is POS [likely that it will rain or snow]⟧ = $\exists d[d > \mathbf{s}_{likely} \wedge Pr(r \vee s) \geq d]$

- The degree of probability of rain or snow is greater than some contextual standard

The account

- With *likely* and *certain* the scope of EXH does not matter.
- Let's assume that POS moves to take scope over EXH leaving a trace of type *d*.

(64) $[\![$POS[EXH[*d* likely that it will rain or snow]]$]\!]$

The account

- With *likely* and *certain* the scope of EXH does not matter.
- Let's assume that POS moves to take scope over EXH leaving a trace of type *d*.

(64) $[\![$POS[EXH[*d* likely that it will rain or snow]]$]\!]$

(65) Alt $= \{ \ldots d$ likely it will rain,
d likely it will snow $\ldots \}$

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- With *likely* and *certain* the scope of EXH does not matter.
- Let's assume that POS moves to take scope over EXH leaving a trace of type d.

(64) $[\![\text{POS}[\text{EXH}[d \text{ likely that it will rain or snow}]]]\!]$

(65) $\text{Alt} = \{\ \ldots d \text{ likely it will rain},$
$d \text{ likely it will snow} \ldots\}$

(66) $Pr(r \vee s) \geq d \wedge \neg[Pr(r) \geq d] \wedge \neg[Pr(s) \geq d] =$

The account

- With *likely* and *certain* the scope of EXH does not matter.
- Let's assume that POS moves to take scope over EXH leaving a trace of type d.

(64) $[\![$POS[EXH[d likely that it will rain or snow]]$]\!]$

(65) Alt $= \{ \ldots d$ likely it will rain, d likely it will snow $\ldots \}$

(66) $Pr(r \vee s) \geq d \wedge \neg[Pr(r) \geq d] \wedge \neg[Pr(s) \geq d] = Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d$

The account

- Once you compose the meaning with POS you have

(67) $[\![$It is POS[EXH[likely that it will rain or snow]]$]\!] =$
$\exists d[d > \mathbf{s}_{likely} \wedge Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d]$

- This entails that the probability of rain is non-zero and that of snow is non-zero
- But then given our analysis of *possible* the distributive inferences immediately follow

The account

- The reasoning above extends straightforwardly to *certain*

The account

- consider the case of *certain*

(68) $[\![\text{POS}[\text{EXH}[d$ certain that it will rain or snow$]]]\!]$

The account

- consider the case of *certain*

(68) $[\![$POS[EXH[*d* certain that it will rain or snow]]$]\!]$

(69) Alt = { . . . *d* certain it will rain,
d certain it will snow . . . }

The account

- consider the case of *certain*

(68) $[\![\text{POS}[\text{EXH}[d \text{ certain that it will rain or snow}]]]\!]$

(69) Alt = { . . . *d* certain it will rain,
d certain it will snow . . . }

(70) $Pr(r \vee s) \geq d \wedge \neg[Pr(r) \geq d] \wedge \neg[Pr(s) \geq d] =$

The account

- consider the case of *certain*

(68) $[\![\text{POS}[\text{EXH}[d \text{ certain that it will rain or snow}]]]\!]$

(69) Alt $= \{ \ldots d$ certain it will rain,
d certain it will snow $\ldots \}$

(70) $Pr(r \vee s) \geq d \wedge \neg[Pr(r) \geq d] \wedge \neg[Pr(s) \geq d] =$
$Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d$

The account

- Then again you compose the meaning with POS

(71) $[\![$It is POS[EXH[certain that it will rain or snow]]$]\!] =$
$\exists d[d = 1 \wedge Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d]$

The account

- Then again you compose the meaning with POS

(71) $[\![$It is POS[EXH[certain that it will rain or snow]]$]\!] =$
$\exists d[d = 1 \wedge Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d]$

- (71) entails that the probability of rain is non-zero and that of snow is non-zero

The account

- Then again you compose the meaning with POS

(71) $[\![$It is POS[EXH[certain that it will rain or snow]]$]\!] =$
$\exists d[d = 1 \wedge Pr(r \vee s) \geq d \wedge Pr(r) < d \wedge Pr(s) < d]$

- (71) entails that the probability of rain is non-zero and that of snow is non-zero
- And again given our analysis of *possible* the distributive inferences immediately follow

The account

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The account

- Once you compose the meaning with POS you have

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- And again given our analysis of *possible* the free choice inferences immediately follow

In sum

- We can account for both distributive and free choice inferences via the same mechanism
- By treating epistemic modal adjectives as gradable adjectives and have EXH apply below POS

Two main points for discussion

1. How this account extends beyond epistemic adjectives
2. How to respond to skepticism about a probabilistic implementation and the gradability of (some of) these adjectives

Outline

Modal auxiliaries

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- The question is whether we can plausibly separate the degree element from the position in which existential closure would happen
- So that we can insert an exhaustivity operator in between

Modal auxilaries

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(80) It might be that it's raining in Sydney

(81) It might very well be that it's raining in Sydney

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- Can we extend our analysis to these cases?

Beyond epistemic modals

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- The reason: the EU of a disjunction is not, in general, an upper bound on the EUs of the disjuncts.

$$EU(p), EU(q) \not\leq EU(p \vee q)$$

- (This is not a big drawback, since EU semantics for deontic modals are problematic—see e.g. Cariani 2015a,b.)

Beyond epistemic modals

Our suggestion: use a probability scale also for deontic modals.

- Roughly: *It is required that p* means that the probability of p, conditional on one of the deontically 'best' worlds being actualized, is 1.

$$[\![\text{it is } \textsc{pos} \text{ required that } p]\!] = 1 \text{ iff}$$
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$$[\![\text{it is } \textsc{pos} \text{ required that } p]\!] = 1 \text{ iff}$$
$$Pr(p|Best) = 1$$

- The account is similar for deontic adjectives with different force, like *allowed*.

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- This has been defended in the literature recently (Yalcin 2010, Lassiter 2010, 2011, a.o.)
- On the other hand, this approach has been challenged by others (e.g. Klecha 2014)

Modality and gradability

- Two main complaints
  - adjectives like *likely* are gradable but do not employ a probability scale
  - adjectives like *possible* are not gradable altogether

Arguments against probabilistic implementation

- proportional modifiers

(84) (??)It is 60% likely that it will rain

(85) #It it totally/completely likely that it will rain

Responses

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- The problem of *not-possible* entailing *not-likely*

(87) It's not possible that it will rain therefore it isn't likely that it will rain

Arguments against gradable analysis of *possible*

- comparatives and modifiers

(88) ??It's slightly possible that it will rain

(89) ??It's very possible that it will rain

(90) ??It's 30% possible that it will rain

(91) ??It's more possible that it will rain than it will snow

Although

(92) It is slightly possible that the Jets will win. (Lassiter 2010)

(93) I felt that if it was 80-90 percent possible that [the cancer] hadn't spread, I didn't want the hysterectomy. (Lassiter 2011)

(94) In fact, it is more possible that tomorrow is the zombie apocalypse than people magically floating away into the clouds. (Lassiter 2016)

Response

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- On a probabilistic analysis, *possible* is a minimum standard adjectives that exploit a closed scale
- Some items with the same features (from Kennedy & McNally 2005): *acquainted*, *documented*, *understood*

Response

- In English, *Possible* does share modifiers with these items: e.g. *scarcely* and (at least in British English) *well*

(95) It's (very) well possible that Mary will win the race

- Similar situation in Italian, where the relevant intensifier is *ampiamente*

(96) È ampiamente possible che Maria vinca la gara
Is amply possible that Mary win the race

In sum

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- It's at least not obvious that a probabilistic implementation of *likely* and *certain* is problematic
- And it's also not obvious that a gradable analysis of *possible* is problematic
- In fact, there is some evidence for a 'spotty gradability' of *possible* with modifiers like *well* or *ampiamente*

Summary

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- Adopting a degree semantics of epistemic modal adjectieves
- A unified account of their distributive and free choice inferences when they embed a disjunction
- We sketched how the analysis can be extended beyond modal adjectives and beyond epistemic modality
- We defended a gradable analysis of *possible* and the use of a probability scale

Future work

- Exploring an extension of the analysis to plural individuals

(97) Some students finished in three months or didn't finish at all
$\rightsquigarrow$ some students finished in three months and some students didn't finish at all

- Exploring an extension to to Free choice items like *any*

Thanks!

- Collaborator:

- Others:
  Fabrizio Cariani, Danny Fox, Alexis Wellwood, Rick Nouwen

on Crnic et al

-

Comparison with other accounts

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Wide scope disjunction

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