Probabilistic experimental pragmatics beyond Bayes' theorem

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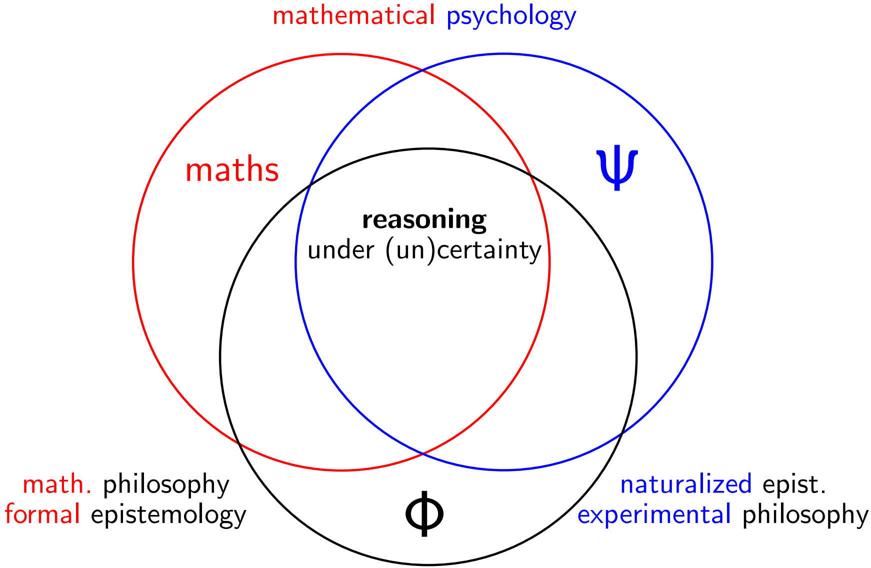
Table of contents

Disciplines



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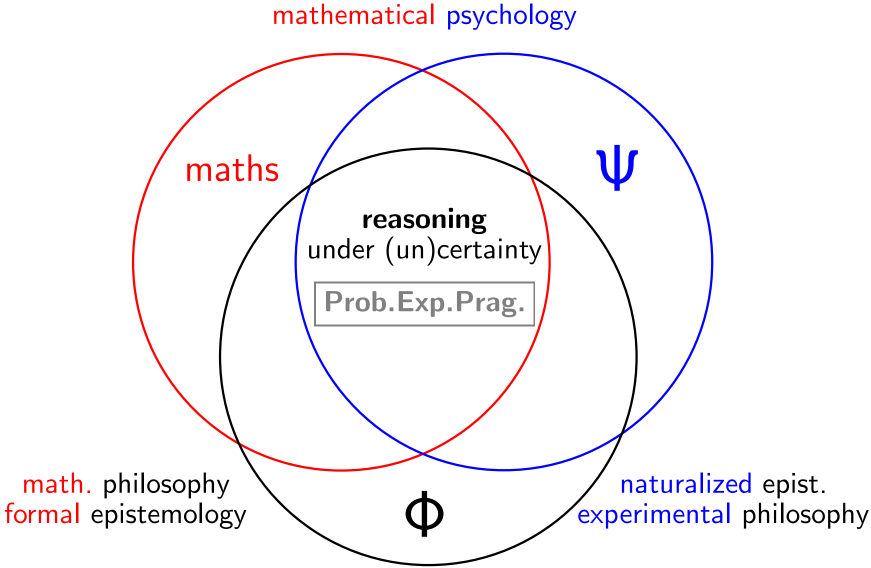

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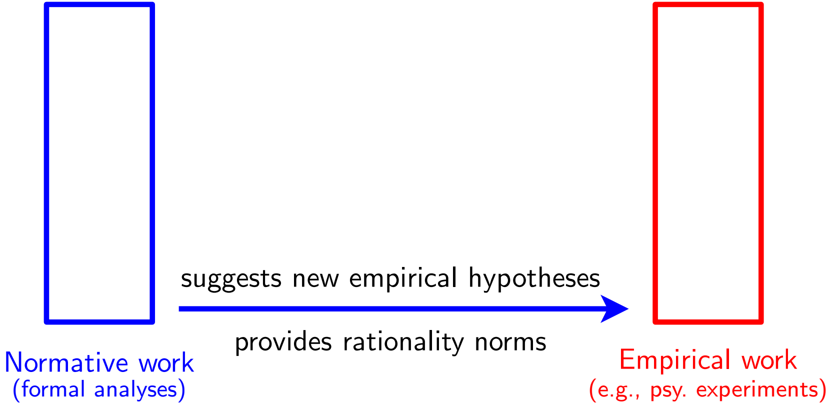

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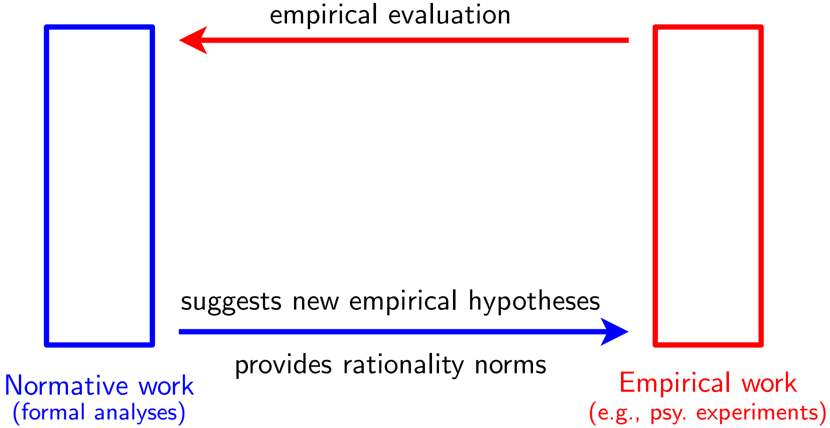

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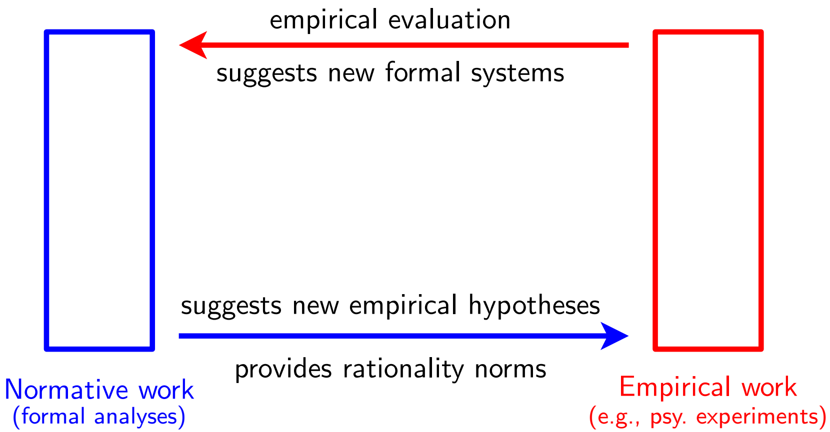

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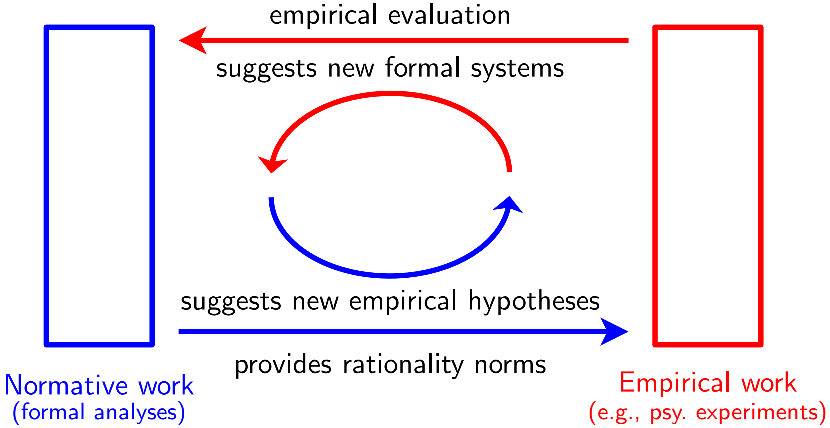

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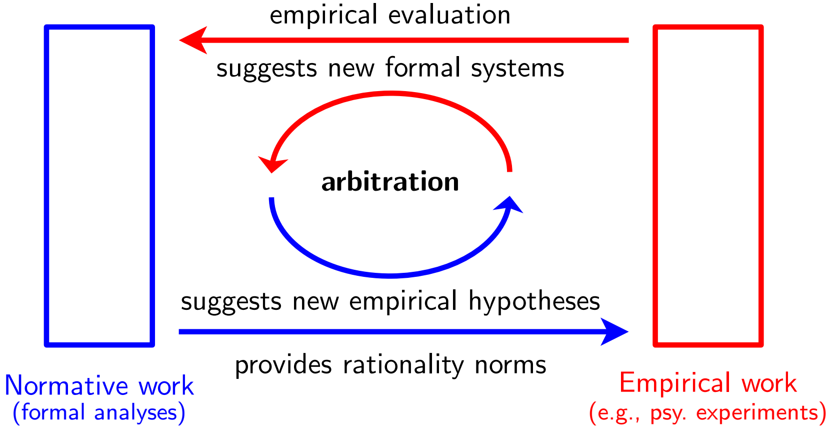

Why not classical logic?

- unable to deal with degrees of belief
- unable to deal with nonmonotonicity
- interpreting natural language conditionals by the material conditional $(\cdot \supset \cdot)$ is highly problematic

Truth tables

Negation:

A	not-A $\neg A$
T	F
F	T

Samples of other connectives:

A	B	A and B $A \wedge B$	A or B $A \vee B$	If A, then B $A \supset B$	A iff B $A \equiv B$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Truth tables & Ramsey test

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Samples of other connectives:

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T	T	T	T	T	T	T
T	F	F	T	F	F	F
F	T	F	T	T	F	void
F	F	F	F	T	T	void

> *"If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; ... We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered void"* (Ramsey, 1929/1994, footnote, p. 155).

Truth tables & Ramsey test

Samples of other connectives:

A	B			If A, then B $A \supset B$		B given A $B \mid A$
T	T			T		T
T	F			F		F
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Table of contents

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  - if no: STOP ($[0, 1]$ is coherent)
  - if yes: transmit the uncertainty from the premises to the conclusion

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- rationality framework: coherence-based probability logic framework

Coherence-based probability logic

- Coherence
  - de Finetti, and $\{$Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...$\}$
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $P(B|A)$ by

$$\frac{P(A \wedge B)}{P(A)} \quad \text{and assume that} \quad P(A) > 0$$

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    what if $P(A) = 0$?

    in the coherence approach, conditional probability, $P(B|A)$, is primitive
  - zero probabilities are exploited to reduce the complexity
  - imprecision
  - bridges to possibility, DS-belief functions, fuzzy sets, nonmonotonic reasoning (System P (Gilio, 2002)), . . .

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- Probability logic
  - uncertain argument forms
  - deductive consequence relation

Bayes' theorem

...as an uncertain argument form:

(Premise 1)	$p(B\|A) = x$
(Premise 2)	$p(A) = y$
(Premise 3)	$p(B) = z$
(Conclusion)	$p(A\|B) = xy/z$

Bayes' theorem

. . . as an uncertain argument form:

$$\begin{array}{ll} \text{(Premise 1)} & p(B|A) = x \\ \text{(Premise 2)} & p(A) = y \\ \text{(Premise 3)} & p(B) = z \\ \hline \text{(Conclusion)} & p(A|B) = xy/z \end{array}$$

. . . as a (probability-logical) rule of inference:

From $p(B|A) = x$, $p(A) = y$, and $p(B) = z$ infer $p(A|B) = xy/z$.

Bayes' theorem

...as an uncertain argument form:

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\text{(Premise 1)} & p(B|A) = x \\
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\text{(Conclusion)} & p(A|B) = xy/z
\end{array}$$

...as a (probability-logical) rule of inference:

From $p(B|A) = x$, $p(A) = y$, and $p(B) = z$ infer $p(A|B) = xy/z$.

Observation: Bayes' theorem is one of many important theorems for "probabilistic experimental pragmatics."

E.g.: Probabilistic modus ponens (e.g., Hailperin, 1996; Pfeifer & Kleiter, 2006a)

Modus ponens	Probabilistic modus ponens	
	(Conditional event)	*(Material conditional)*
If A, then C	$p(C\|A) = x$	$p(A \supset C) = x$
A	$p(A) = y$	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$	$\max\{0, x + y - 1\} \leq p(C) \leq x$

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...where the consequence relation ("———") is deductive.

...interpretation of "if–then" matters!

Example 2: Probabilistic modus ponens (e.g., Hailperin, 1996)

Modus ponens	Probabilistic modus ponens	
	(Conditional event)	*(Material conditional)*
If A, then C	$p(C\|A) = .90$	$p(A \supset C) = .90$
A	$p(A) = .50$	$p(A) = .50$
C	$.45 \leq p(C) \leq .95$	$.40 \leq p(C) \leq .90$

...where the consequence relation ("———") is deductive.

From probability logic to probabilisitic pragmatics

Consider a probability logical argument with n premises:

$$\frac{\begin{array}{l}\text{Premise 1}\\ \dots\\ \text{Premise n}\end{array}}{\text{Conclusion}}$$

From probability logic to probabilisitic pragmatics

Consider a probability logical argument with n premises:

Premise 1	$\Longrightarrow$	...what the speaker says
...	...	...
Premise n	$\Longrightarrow$	...what the speaker says
Conclusion	$\Longrightarrow$	...what the listeners hears/infers

Table of contents

Sample paradoxes of the material conditional

(Paradox 1)

$$\frac{B}{\text{If } A\text{, then } B}$$

(Paradox 2)

$$\frac{\text{Not: } A}{\text{If } A\text{, then } B}$$

Sample paradoxes of the material conditional

(Paradox 1)	(Paradox 2)
B	Not: A
If A, then B	If A, then B

(Paradox 1)	(Paradox 2)
B	$\neg A$
$A \supset B$	$A \supset B$

Sample paradoxes of the material conditional

(Paradox 1)

$$\frac{P(B) = x}{x \leq P(A \supset B) \leq 1}$$

(Paradox 2)

$$\frac{P(\neg A) = x}{1 - x \leq P(A \supset B) \leq 1}$$

probabilistically informative

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Sample paradoxes of the material conditional (Pfeifer, 2014, *Studia Logica*)

Paradoxes of the material conditional, e.g.,

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This matches the data (Pfeifer & Kleiter, 2011).

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Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:

If $P(B) = 1$, then $P(A \wedge B) = P(A)$.

Sample paradoxes of the material conditional (Pfeifer, 2014, *Studia Logica*)

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Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:

If $P(B) = 1$, then $P(A \wedge B) = P(A)$. Thus,

$P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{P(A)}{P(A)} = 1$, if $P(A) > 0$.

Inf. vers. of t. paradoxes (Pfeifer (2014). *Studia Logica*; Pfeifer and Douven (2014). *R*ev. Phil. Psy.)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

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From $\Pr(B) = x$ and $\Pr(A) = y$ infer

$$\max\left\{0, \frac{x+y-1}{y}\right\} \leqslant \Pr(B \mid A) \leqslant \min\left\{\frac{x}{y}, 1\right\} \text{ is coherent.}$$

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$$\max\left\{0, \frac{x + y - 1}{y}\right\} \leqslant \Pr(B \mid A) \leqslant \min\left\{\frac{x}{y}, 1\right\}$$ is coherent.

. . . a special case of the cautious monotonicity rule of System P (Gilio, 2002).

Table of contents

Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} P(A \wedge C) & = & x_1 \\ P(A \wedge \neg C) & = & x_2 \\ P(\neg A \wedge C) & = & x_3 \\ P(\neg A \wedge \neg C) & = & x_4 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

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Conclusion candidates:

- $P(A \wedge C) = x_1$
- $P(C|A) = x_1/(x_1 + x_2)$
- $P(A \supset C) = x_1 + x_3 + x_4$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

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\hline
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P(\text{If } A, \text{ then } C) & = & ?
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Conclusion candidates:

- $P(A \wedge C) = x_1 = .25$
- $P(C|A) = x_1/(x_1 + x_2) = .50$
- $P(A \supset C) = x_1 + x_3 + x_4 = .75$

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Main results:

- More than half of the responses are consistent with $P(C|A)$
- Many responses are consistent with $P(A \wedge C)$

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Key feature:

- Reasoning under complete probabilistic knowledge

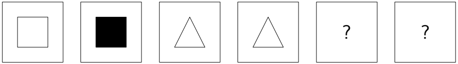
Experiment

Motivation

- probabilistic truth table task with incomplete probabilistic knowledge
- Is the conditional event interpretation still dominant?
- Are there shifts of interpretation?

Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



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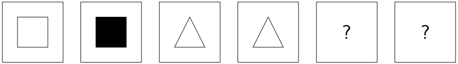

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

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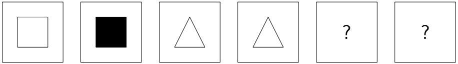
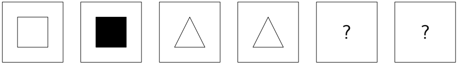


Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer:

at least

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
		1	2	3	4	5	6

at most

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
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(please tick the appropriate boxes)

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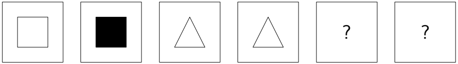

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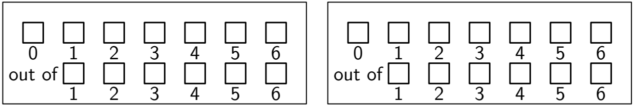
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Cond. event: *at least* 1 out of 5 and *at most* 3 out of 5

at least *at most*



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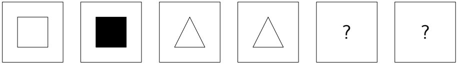

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Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Conjunction: *at least* 1 out of 6 and *at most* 3 out of 6

at least

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
		1	2	3	4	5	6

at most

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
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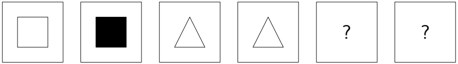

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Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Mat. cond.: *at least* 2 out of 6 and *at most* 4 out of 6

at least

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
		1	2	3	4	5	6

at most

	☐	☐	☐	☐	☐	☐	☐
	0	1	2	3	4	5	6
out of		☐	☐	☐	☐	☐	☐
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Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

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Sample

- 20 Cambridge University students
- 10 female, 10 male
- between 18 and 27 years old (mean: 21.65)
- no students of mathematics, philosophy, computer science, or psychology

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

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Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
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Results

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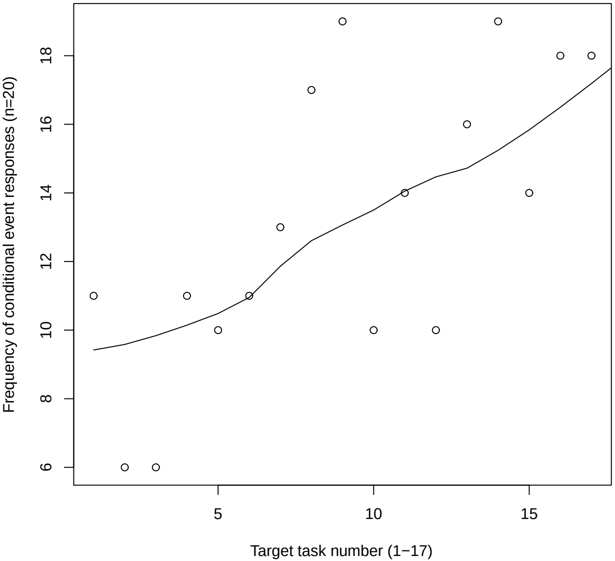
  - 0.3% consistent with material conditional
- Shift of interpretation
  - First three tasks: 38.3% consistent with conditional event
  - Last three tasks: 83.3% consistent with conditional event
  - Strong correlation between conditional event frequency and item position ($r(15) = 0.71$, $p < 0.005$)

Increase of cond. event resp. ($n_1 = 20$) (Pfeifer, 2013a, *Thinking & Reasoning*)



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  Most people judge (correctly) $p(even|x = 2) = 1$
  but (incorrectly) $p(x = 2 \vee x = 4|x = 2) = 0$
  (Fugard, Pfeifer, & Mayerhofer, 2011).

Table of contents

The Tweety problem

The Tweety problem (picture© by L. Ewing, S. Budig, A. Gerwinski; http://commons.wikimedia.org)

The Tweety problem (picture© by ytse19; http://mi9.com/flying-tux_35453.html)

System P: Rationality postulates for nonmonotonic reasoning (Kraus, Lehmann, & Magidor, 1990)

Reflexivity (axiom): $\alpha \mid\!\sim \alpha$

Left logical equivalence:

from $\models \alpha \equiv \beta$ and $\alpha \mid\!\sim \gamma$ infer $\beta \mid\!\sim \gamma$

Right weakening:

from $\models \alpha \supset \beta$ and $\gamma \mid\!\sim \alpha$ infer $\gamma \mid\!\sim \beta$

Or: from $\alpha \mid\!\sim \gamma$ and $\beta \mid\!\sim \gamma$ infer $\alpha \vee \beta \mid\!\sim \gamma$

Cut: from $\alpha \wedge \beta \mid\!\sim \gamma$ and $\alpha \mid\!\sim \beta$ infer $\alpha \mid\!\sim \gamma$

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$\alpha \mid\!\sim \beta$ is read as If α, $\underbrace{\textit{normally}}_{?}$ β

Probabilistic version of System P (Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

Name	*Probability logical version*
Left logical equivalence	$\models(E_1 \equiv E_2), P(E_3\|E_1) = x \therefore P(E_3\|E_2) = x$
Right weakening	$P(E_1\|E_3) = x, \models(E_1 \supset E_2) \therefore P(E_2\|E_3) \in [x, 1]$
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Transitivity	$P(E_2\|E_1) = x, P(E_3\|E_2) = y \therefore P(E_3\|E_1) \in [0, 1]$
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. . . probabilistically non-informative

The Tweety problem (Pfeifer, 2012b)

$\mathfrak{P}1$	$P[\text{Fly}(x)\vert\text{Bird}(x)] = .95.$	*(Birds can normally fly.)*
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$\mathfrak{C}2$ $P[\text{Fly}(\text{Tweety}) \mid \text{Bird}(\text{Tweety}) \wedge \text{Penguin}(\text{Tweety})] \in [0, .01].$
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- Experimental result: Non-probabilistic tasks: endorsement rate of 89–100% (Evans et al., 1993); probabilistic tasks: 63%-100% coherent responses (Pfeifer & Kleiter, 2007)

The Transitivity Task

(Pfeifer & Kleiter, 2006b)

Please imagine the following situation:

Exactly 99% of the cars on a big parking lot are blue.
Exactly 63% of blue cars have grey wheel rims.

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Please imagine the following situation:

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Imagine all the cars that are on the big parking lot. How many of these cars have grey wheel rims?

The Transitivity Task interpreted as Cut (Pfeifer & Kleiter, 2006b)

Please imagine the following situation:

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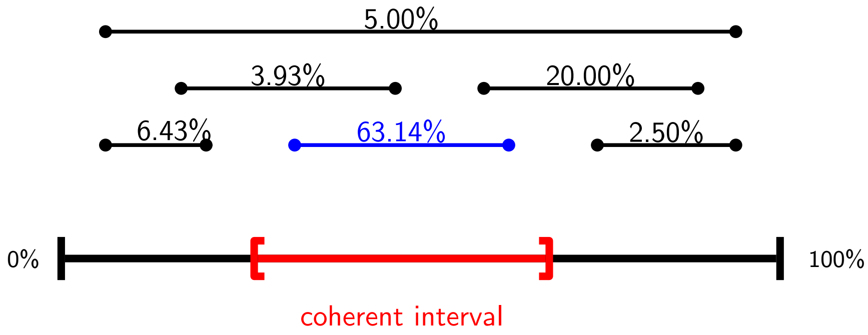
have grey wheel rims.

Imagine all the cars that are on the big parking lot. How many of these cars have grey wheel rims?

(Adams, 1975; Bennett, 2003)

Results: Transitivity. . . "as Cut" (Pfeifer & Kleiter, 2006b)



averaged interval response frequencies, 14 tasks, $n = 20$

Table of contents

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- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret conditionals by conditional probabilities:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
- Most people draw coherent inferences.

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Concluding remarks

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- Premises represent what the speaker says and conclusions represent what the hearer infers
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- Long term goal: Theory of uncertain inference which is normatively and descriptively adequate

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Appendix

Properties of arguments

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- An argument is logically valid if and only if it is impossible that all premises are true and the conclusion is false.

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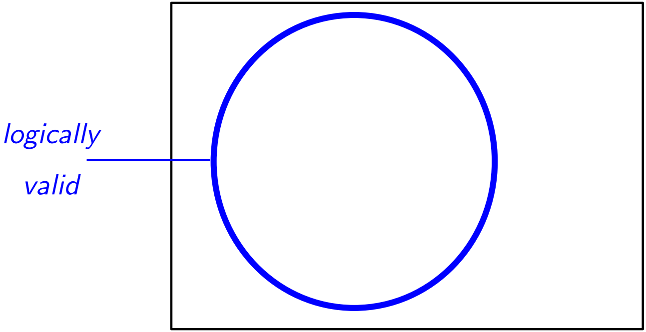
- An argument is probabilistically informative if and only if it is possible that the premise probabilities constrain the conclusion probability. I.e., if the coherent probability interval of its conclusion is not necessarily equal to the unit interval $[0, 1]$ (Pfeifer & Kleiter, 2006a).

Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)



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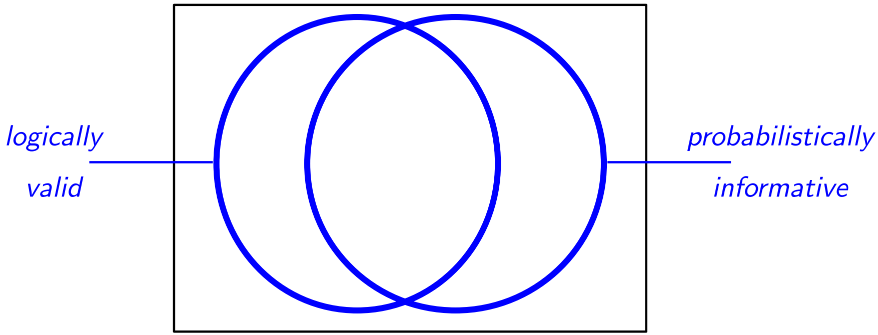

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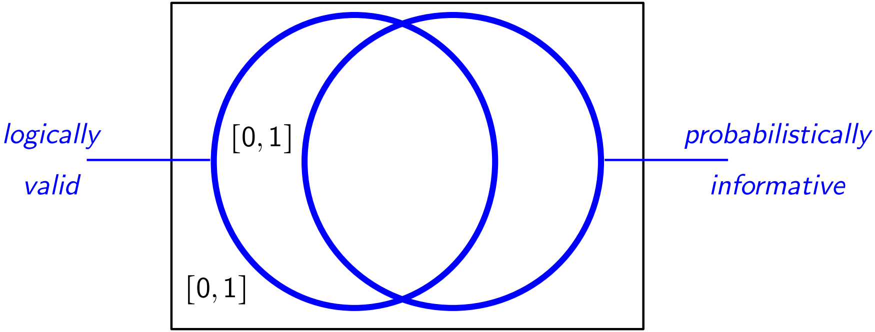

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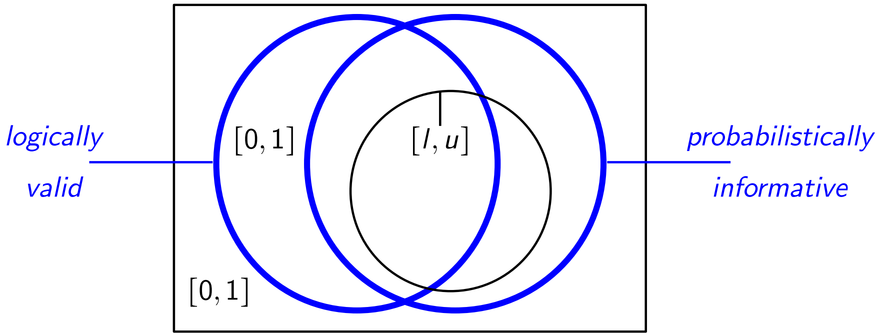

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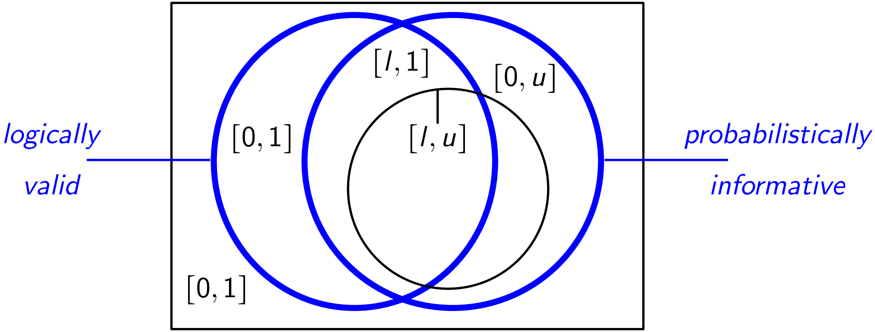

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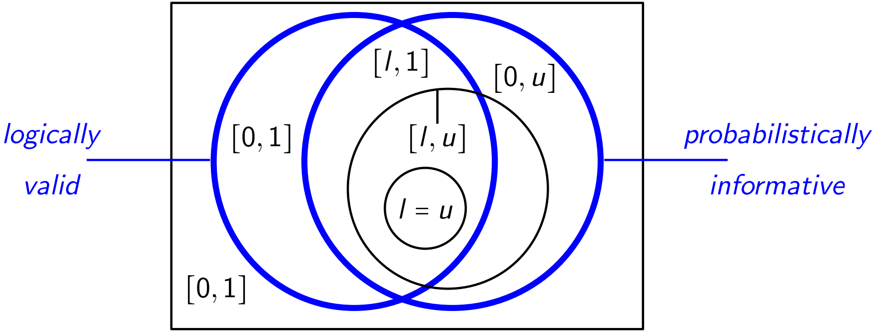

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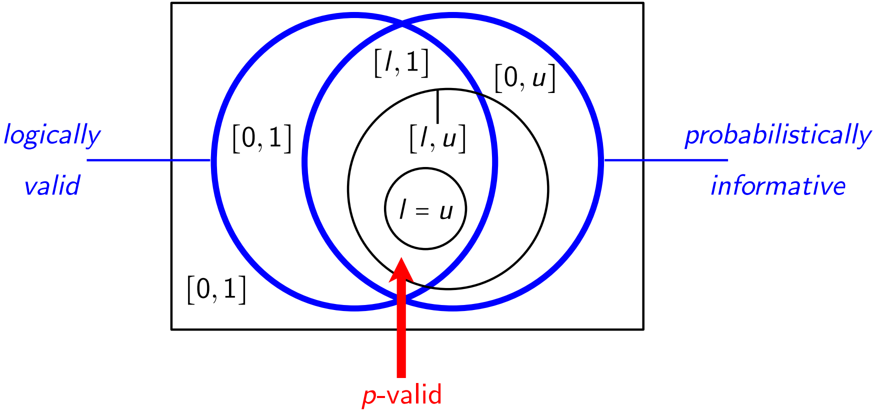

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Example 1: (Cautious) monotonicity

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{(A \wedge C) \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(B|A \wedge C) \leq 1}$

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  But: from $\boxed{P(A \supset B) = x}$ infer $\boxed{x \leq P((A \wedge C) \supset B) \leq 1}$

Example 1: (Cautious) monotonicity

- In logic

  from $A \supset B$ infer $(A \wedge C) \supset B$

- In probability logic

  from $P(B|A) = x$ infer $0 \leq P(B|A \wedge C) \leq 1$

  But: from $P(A \supset B) = x$ infer $x \leq P((A \wedge C) \supset B) \leq 1$

- Cautious monotonicity (Gilio, 2002)

  from $P(B|A) = x$ and $P(C|A) = y$

  infer $\max(0, (x + y - 1)/x) \leq P(C|A \wedge B) \leq \min(y/x, 1)$

Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

exactly 72% wear a black suit.

Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

exactly 72% wear a black suit.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

Example task: Cautious monotonicity (Pfeifer & Kleiter, 2003)

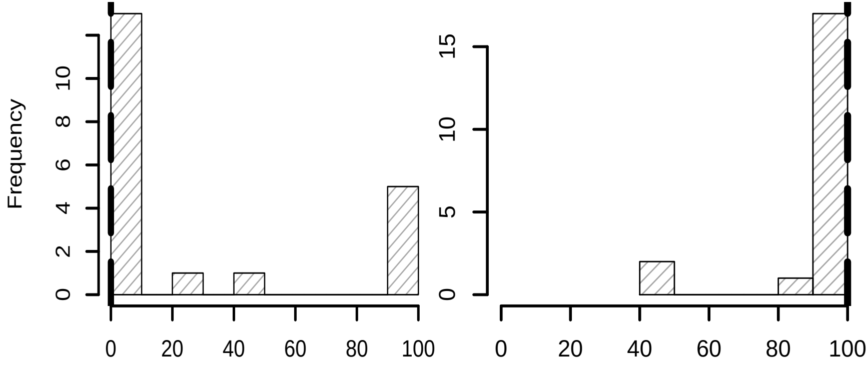
About the guests at a prom we know the following:

exactly 72% wear a black suit.
exactly 63% wear glasses.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

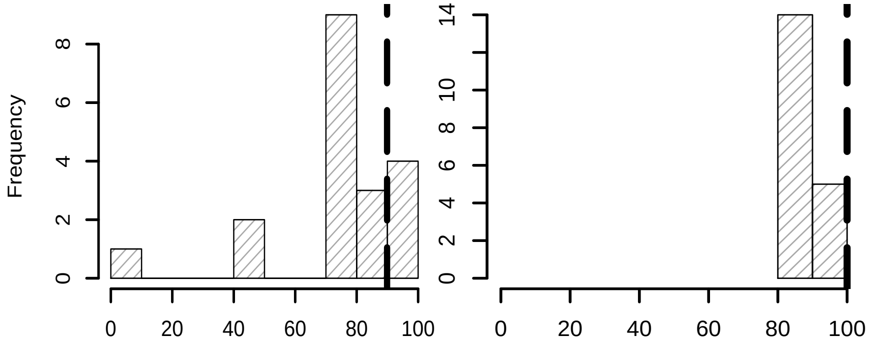
Results – Monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

$(n_1 = 20)$

Results – Cautious monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

$(n_2 = 19)$

Example 2: Contraposition

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{\neg B \supset \neg A}$

  from $\boxed{\neg B \supset \neg A}$ infer $\boxed{A \supset B}$

Example 2: Contraposition

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{\neg B \supset \neg A}$

  from $\boxed{\neg B \supset \neg A}$ infer $\boxed{A \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(\neg A|\neg B) \leq 1}$

  from $\boxed{P(\neg A|\neg B) = x}$ infer $\boxed{0 \leq P(B|A) \leq 1}$

Example 2: Contraposition

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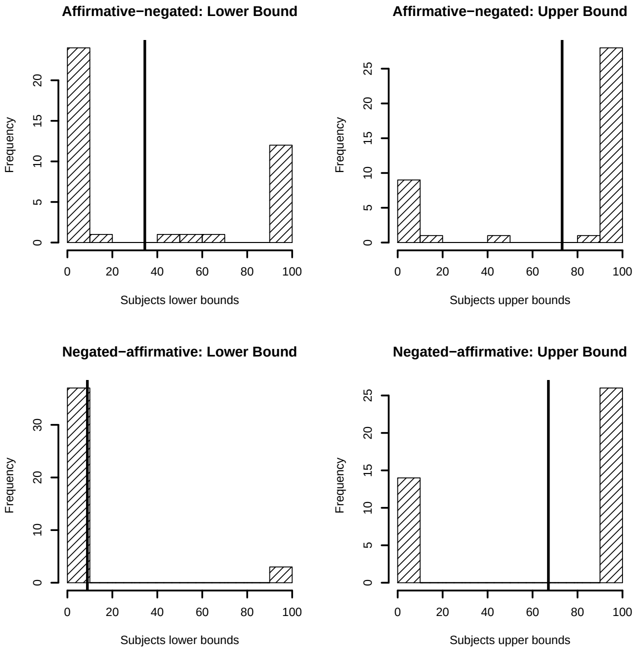
  from $\boxed{\neg B \supset \neg A}$ infer $\boxed{A \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(\neg A|\neg B) \leq 1}$

  from $\boxed{P(\neg A|\neg B) = x}$ infer $\boxed{0 \leq P(B|A) \leq 1}$
- But

$$P(A \supset B) = P(\neg B \supset \neg A)$$

Results Contraposition ($n_1 = 40$, $n_2 = 40$; Pfeifer and Kleiter (2006b))



References I

Adams, E. W. (1975). *The logic of conditionals. An application of probability to deduction*. Dordrecht: Reidel.

Bennett, J. (2003). *A philosophical guide to conditionals*. Oxford: Oxford University Press.

Evans, J. St. B. T., Handley, S. J., & Over, D. E. (2003). Conditionals and conditional probability. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *29*(2), 321-355.

Evans, J. St. B. T., Newstead, S. E., & Byrne, R. M. J. (1993). *Human reasoning. The psychology of deduction*. Hove: Lawrence Erlbaum.

Fugard, A. J. B., Pfeifer, N., & Mayerhofer, B. (2011). Probabilistic theories of reasoning need pragmatics too: Modulating relevance in uncertain conditionals. *Journal of Pragmatics*, *43*, 2034-2042.

References II

Fugard, A. J. B., Pfeifer, N., Mayerhofer, B., & Kleiter, G. D. (2011a). How people interpret conditionals: Shifts towards the conditional event. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *37*(3), 635–648.

Gilio, A. (2002). Probabilistic reasoning under coherence in System P. *Annals of Mathematics and Artificial Intelligence*, *34*, 5-34.

Hailperin, T. (1996). *Sentential probability logic. Origins, development, current status, and technical applications*. Bethlehem: Lehigh University Press.

Kraus, S., Lehmann, D., & Magidor, M. (1990). Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence*, *44*, 167-207.

Oberauer, K., & Wilhelm, O. (2003). The meaning(s) of conditionals: Conditional probabilities, mental models and personal utilities. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *29*, 680–693.

References III

Pfeifer, N. (2006a). Contemporary syllogistics: Comparative and quantitative syllogisms. In G. Kreuzbauer & G. J. W. Dorn (Eds.), *Argumentation in Theorie und Praxis: Philosophie und Didaktik des Argumentierens* (p. 57-71). Wien: LIT Verlag.

Pfeifer, N. (2006b). *On mental probability logic* (Unpublished doctoral dissertation). Department of Psychology, University of Salzburg. (The abstract is published in *The Knowledge Engineering Review*, 2008, 23, pp. 217-226; `http://www.pfeifer-research.de/pdf/diss.pdf`)

Pfeifer, N. (2007). Rational argumentation under uncertainty. In G. Kreuzbauer, N. Gratzl, & E. Hiebl (Eds.), *Persuasion und Wissenschaft: Aktuelle Fragestellungen von Rhetorik und Argumentationstheorie* (p. 181-191). Wien: LIT Verlag.

References IV

Pfeifer, N. (2008). A probability logical interpretation of fallacies. In G. Kreuzbauer, N. Gratzl, & E. Hiebl (Eds.), *Rhetorische Wissenschaft: Rede und Argumentation in Theorie und Praxis* (pp. 225–244). Wien: Lit Verlag.

Pfeifer, N. (2010, February). *Human conditional reasoning and Aristotle's Thesis.* Talk. PROBNET'10 (Probabilistic networks) workshop, Salzburg (Austria).

Pfeifer, N. (2011). Systematic rationality norms provide research roadmaps and clarity. Commentary on Elqayam & Evans: Subtracting "ought" from "is": Descriptivism versus normativism in the study of human thinking. *Behavioral and Brain Sciences*, *34*, 263–264.

Pfeifer, N. (2012a). Experiments on Aristotle's Thesis: Towards an experimental philosophy of conditionals. *The Monist*, *95*(2), 223–240.

References V

Pfeifer, N. (2012b). *Naturalized formal epistemology of uncertain reasoning* (Unpublished doctoral dissertation). Tilburg Center for Logic and Philosophy of Science, Tilburg University.

Pfeifer, N. (2013a). The new psychology of reasoning: A mental probability logical perspective. *Thinking & Reasoning*, *19*(3–4), 329–345.

Pfeifer, N. (2013b). On argument strength. In F. Zenker (Ed.), *Bayesian argumentation. The practical side of probability* (pp. 185–193). Dordrecht: Synthese Library (Springer).

Pfeifer, N. (2014). Reasoning about uncertain conditionals. *Studia Logica*, *102*(4), 849-866.

Pfeifer, N., & Douven, I. (2014). Formal epistemology and the new paradigm psychology of reasoning. *The Review of Philosophy and Psychology*, *5*(2), 199–221.

References VI

Pfeifer, N., & Kleiter, G. D. (2003). Nonmonotonicity and human probabilistic reasoning. In *Proceedings of the 6th workshop on uncertainty processing* (p. 221-234). Hejnice: September 24–27, 2003.

Pfeifer, N., & Kleiter, G. D. (2005a). Coherence and nonmonotonicity in human reasoning. *Synthese*, *146*(1-2), 93-109.

Pfeifer, N., & Kleiter, G. D. (2005b). Towards a mental probability logic. *Psychologica Belgica*, *45*(1), 71-99.

Pfeifer, N., & Kleiter, G. D. (2006a). Inference in conditional probability logic. *Kybernetika*, *42*, 391-404.

Pfeifer, N., & Kleiter, G. D. (2006b). Is human reasoning about nonmonotonic conditionals probabilistically coherent? In *Proceedings of the 7th workshop on uncertainty processing* (p. 138-150). Mikulov: September 16–20, 2006.

References VII

Pfeifer, N., & Kleiter, G. D. (2007). Human reasoning with imprecise probabilities: Modus ponens and Denying the antecedent. In G. De Cooman, J. Vejnarová, & M. Zaffalon (Eds.), *Proceedings of the 5th international symposium on imprecise probability: Theories and applications* (p. 347-356). Prague: SIPTA.

Pfeifer, N., & Kleiter, G. D. (2009). Framing human inference by coherence based probability logic. *Journal of Applied Logic*, *7*(2), 206–217.

Pfeifer, N., & Kleiter, G. D. (2010). The conditional in mental probability logic. In M. Oaksford & N. Chater (Eds.), *Cognition and conditionals: Probability and logic in human thought* (pp. 153–173). Oxford: Oxford University Press.

Pfeifer, N., & Kleiter, G. D. (2011). Uncertain deductive reasoning. In K. Manktelow, D. E. Over, & S. Elqayam (Eds.), *The science of reason: A Festschrift for Jonathan St. B.T. Evans* (p. 145-166). Hove: Psychology Press.

References VIII

Pfeifer, N., & Stöckle-Schobel, R. (2015). Uncertain conditionals and counterfactuals in (non-)causal settings. In G. Arienti, B. G. Bara, & S. G. (Eds.), *Proceedings of the Euroasianpacific joint conference on cognitive science (4th European conference on cognitive science; 10th international conference on cognitive science)* (Vol. 1419, pp. 651–656). Aachen: CEUR Workshop Proceedings. Retrieved from `http://ceur-ws.org/Vol-1419/paper0108.pdf`

Ramsey, F. P. (1929/1994). General propositions and causality (1929). In D. H. Mellor (Ed.), *Philosophical papers by F. P. Ramsey* (p. 145-163). Cambridge: Cambridge University Press.