Probabilistic experimental pragmatics beyond Bayes’ theorem

Niki Pfeifer\textsuperscript{1}

Munich Center for Mathematical Philosophy (LMU Munich)

\textsuperscript{1}Supported by the German Research Foundation (project grants PF 740/2-1 and PF 740/2-2 within the Priority Program SPP 1516 “New Frameworks of Rationality”).
Introduction
   Disciplines & interaction of normative and empirical work
   Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks
References
Disciplines

- mathematical psychology
- maths
- reasoning under (un)certainty
  - Prob.Exp.Prág.
- math. philosophy
- formal epistemology
- naturalized epist. experimental philosophy
Interaction of normative and empirical work (Pfeifer, 2011, 2012b)

Normative work (formal analyses)

Empirical work (e.g., psy. experiments)
Interaction of normative and empirical work  
(Pfeifer, 2011, 2012b)

Normative work  
(formal analyses)

suggests new empirical hypotheses

provides rationality norms

Empirical work  
(e.g., psy. experiments)
Interaction of normative and empirical work (Pfeifer, 2011, 2012b)

Normative work (formal analyses)

suggests new empirical hypotheses

provides rationality norms

empirical evaluation

Empirical work (e.g., psy. experiments)
Interaction of normative and empirical work (Pfeifer, 2011, 2012b)

Normative work (formal analyses)

Empirical work (e.g., psy. experiments)

empirical evaluation
suggests new formal systems

suggests new empirical hypotheses

provides rationality norms
Interaction of normative and empirical work

(Pfeifer, 2011, 2012b)

Normative work
(formal analyses)

Empirical work
(e.g., psy. experiments)

- Provides rationality norms
- Suggests new empirical hypotheses
- Empirical evaluation
  - Suggests new formal systems
Interaction of normative and empirical work  
(Pfeifer, 2011, 2012b)

Normative work (formal analyses)

empirical evaluation
suggests new formal systems

empirical evaluation
suggests new empirical hypotheses

provides rationality norms

arbitration

Empirical work (e.g., psy. experiments)
Why not classical logic?

- unable to deal with degrees of belief
- unable to deal with nonmonotonicity
- interpreting natural language conditionals by the material conditional ($\cdot \supset \cdot$) is highly problematic
### Truth tables

**Negation:**

<table>
<thead>
<tr>
<th></th>
<th>not-(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

**Samples of other connectives:**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>A and B</th>
<th>A or B</th>
<th>If A, then B</th>
<th>A iff B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>(A \land B)</td>
<td>(A \lor B)</td>
<td>(A \supset B)</td>
<td>(A \equiv B)</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
**Truth tables & Ramsey test**

**Negation:**

<table>
<thead>
<tr>
<th>$A$</th>
<th>not-$A$</th>
<th>$\neg A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

**Samples of other connectives:**

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$A$ and $B$</th>
<th>$A$ or $B$</th>
<th>If $A$, then $B$</th>
<th>$A$ iff $B$</th>
<th>$B$ given $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>void</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>void</td>
</tr>
</tbody>
</table>

“*If two people are arguing ‘If $p$ will $q$?’ and are both in doubt as to $p$, they are adding $p$ hypothetically to their stock of knowledge and arguing on that basis about $q$; . . . We can say they are fixing their degrees of belief in $q$ given $p$. If $p$ turns out false, these degrees of belief are rendered void*” (Ramsey, 1929/1994, footnote, p. 155).
**Truth tables & Ramsey test**

Samples of other connectives:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A ∨ B</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>------</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>B given A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>void</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>void</td>
</tr>
</tbody>
</table>

“If two people are arguing ‘If p will q?’ and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; . . . We can say they are fixing their degrees of belief in q given p. *If p turns out false, these degrees of belief are rendered void*” (Ramsey, 1929/1994, footnote, p. 155).
Table of contents

Introduction
  Disciplines & interaction of normative and empirical work
  Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks

References

- competence

- competence
- uncertain indicative If A, then C is interpreted as $P(C|A)$

- competence
- uncertain indicative If $A$, then $C$ is interpreted as $P(C|A)$
- $C|A$ is partially truth-functional (void, if $A$ is false and undefined if $A$ is a logical contradiction)

- competence
- uncertain indicative If $A$, then $C$ is interpreted as $P(C|A)$
- $C|A$ is partially truth-functional (void, if $A$ is false and undefined if $A$ is a logical contradiction)
- arguments: $\langle$ premise(s) $, \text{ conclusion} \rangle$

- competence
- uncertain indicative If $A$, then $C$ is interpreted as $P(C|A)$
- $C|A$ is partially truth-functional (void, if $A$ is false and undefined if $A$ is a logical contradiction)
- arguments:  ⟨ premise(s) , conclusion ⟩
- premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)

- competence
- uncertain indicative If $A$, then $C$ is interpreted as $P(C|A)$
- $C|A$ is partially truth-functional (void, if $A$ is false and undefined if $A$ is a logical contradiction)
- arguments: $\langle$ premise(s), conclusion $\rangle$
- premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)
- uncertainty is transmitted deductively from the premises to the conclusion

- **competence**
- uncertain indicative If \( A \), then \( C \) is interpreted as \( P(C|A) \)
- \( C|A \) is partially truth-functional (void, if \( A \) is false and undefined if \( A \) is a logical contradiction)
- **arguments**: \( \langle \text{premise(s)}, \text{conclusion} \rangle \)
- premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)
- uncertainty is transmitted **deductively** from the premises to the conclusion
- **mental process**: check if argument is probabilistically informative
  - if no: STOP ([0, 1] is coherent)
  - if yes: transmit the uncertainty from the premises to the conclusion

- competence
- uncertain indicative If $A$, then $C$ is interpreted as $P(C|A)$
- $C|A$ is partially truth-functional (void, if $A$ is false and undefined if $A$ is a logical contradiction)
- arguments: $\langle$ premise(s) , conclusion $\rangle$
- premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)
- uncertainty is transmitted deductively from the premises to the conclusion
- mental process: check if argument is probabilistically informative
  - if no: STOP ($[0, 1]$ is coherent)
  - if yes: transmit the uncertainty from the premises to the conclusion
- rationality framework: coherence-based probability logic framework
Coherence-based probability logic

- Coherence
  - de Finetti, and \{Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...\}
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define \( P(B|A) \) by

\[
\frac{P(A \land B)}{P(A)} \quad \text{and assume that} \quad P(A) > 0
\]
Coherence-based probability logic

» Coherence
  » de Finetti, and \{Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, \ldots\}
  » degrees of belief
  » complete algebra is not required
  » many probabilistic approaches define $P(B|A)$ by

\[
\frac{P(A \land B)}{P(A)} \quad \text{and assume that} \quad P(A) > 0
\]

what if $P(A) = 0$?
Coherence-based probability logic

- **Coherence**
  - de Finetti, and \{Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, \ldots\}
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $P(B|A)$ by
    \[
    \frac{P(A \land B)}{P(A)}
    \]
    and assume that $P(A) > 0$

  \[\text{what if } P(A) = 0?\]
  in the coherence approach, conditional probability, $P(B|A)$, is primitive

- zero probabilities are exploited to reduce the complexity
- imprecision
- bridges to possibility, DS-belief functions, fuzzy sets, nonmonotonic reasoning (System P (Gilio, 2002)), \ldots
Coherence-based probability logic

- Coherence
  - de Finetti, and {Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...}
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $P(B|A)$ by
    \[
    \frac{P(A \land B)}{P(A)}
    \]
    and assume that $P(A) > 0$

  what if $P(A) = 0$?
  - in the coherence approach, conditional probability, $P(B|A)$, is primitive
  - zero probabilities are exploited to reduce the complexity
  - imprecision
  - bridges to possibility, DS-belief functions, fuzzy sets, nonmonotonic reasoning (System P (Gilio, 2002)), ...

- Probability logic
  - uncertain argument forms
  - deductive consequence relation
Bayes’ theorem

...as an uncertain argument form:

(Premise 1) \( p(B|A) = x \)
(Premise 2) \( p(A) = y \)
(Premise 3) \( p(B) = z \)
(Conclusion) \( p(A|B) = \frac{xy}{z} \)
Bayes’ theorem

...as an uncertain argument form:

(Premise 1) \( p(B|A) = x \)
(Premise 2) \( p(A) = y \)
(Premise 3) \( p(B) = z \)
(Conclusion) \( p(A|B) = \frac{xy}{z} \)

...as a (probability-logical) rule of inference:

From \( p(B|A) = x \), \( p(A) = y \), and \( p(B) = z \) infer \( p(A|B) = \frac{xy}{z} \).
Bayes’ theorem

...as an uncertain argument form:

(Premise 1) \( p(B|A) = x \)
(Premise 2) \( p(A) = y \)
(Premise 3) \( p(B) = z \)
(Conclusion) \( p(A|B) = \frac{xy}{z} \)

...as a (probability-logical) rule of inference:

From \( p(B|A) = x \), \( p(A) = y \), and \( p(B) = z \) infer \( p(A|B) = \frac{xy}{z} \).

Observation: Bayes’ theorem is one of many important theorems for “probabilistic experimental pragmatics.”
E.g.: Probabilistic modus ponens (e.g., Hailperin, 1996; Pfeifer & Kleiter, 2006a)

<table>
<thead>
<tr>
<th>Modus ponens</th>
<th>Probabilistic modus ponens</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $A$, then $C$</td>
<td>$(Conditional$ $event)$</td>
</tr>
<tr>
<td>$A$</td>
<td>$p(C</td>
</tr>
<tr>
<td>$C$</td>
<td>$p(A) = y$</td>
</tr>
<tr>
<td></td>
<td>$xy \leq p(C) \leq xy + 1 - x$</td>
</tr>
<tr>
<td></td>
<td>$max{0, x + y - 1} \leq p(C) \leq x$</td>
</tr>
</tbody>
</table>
E.g.: Probabilistic modus ponens (e.g., Hailperin, 1996; Pfeifer & Kleiter, 2006a)

<table>
<thead>
<tr>
<th>Modus ponens</th>
<th>Probabilistic modus ponens</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $A$, then $C$</td>
<td>(Conditional event)</td>
</tr>
<tr>
<td>$A$</td>
<td>$p(C</td>
</tr>
<tr>
<td>$C$</td>
<td>$p(A) = y$</td>
</tr>
<tr>
<td>$xy \leq p(C) \leq xy + 1 - x$</td>
<td>(Material conditional)</td>
</tr>
<tr>
<td>$p(A \supset C) = x$</td>
<td>$p(A) = y$</td>
</tr>
<tr>
<td>$\max{0, x + y - 1} \leq p(C) \leq x$</td>
<td></td>
</tr>
</tbody>
</table>

...where the consequence relation ("——") is deductive.
E.g.: Probabilistic modus ponens (e.g., Hailperin, 1996; Pfeifer & Kleiter, 2006a)

<table>
<thead>
<tr>
<th>Modus ponens</th>
<th>Probabilistic modus ponens</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $A$, then $C$</td>
<td>Probabilistic modus ponens</td>
</tr>
<tr>
<td>$A$</td>
<td>$(\text{Conditional event})$</td>
</tr>
<tr>
<td>$C$</td>
<td>$p(C</td>
</tr>
<tr>
<td></td>
<td>$p(A) = y$</td>
</tr>
<tr>
<td></td>
<td>$xy \leq p(C) \leq xy + 1 - x$</td>
</tr>
</tbody>
</table>

...where the consequence relation ("——") is deductive.

...interpretation of “if–then” matters!
Example 2: Probabilistic modus ponens (e.g., Hailperin, 1996)

<table>
<thead>
<tr>
<th>Modus ponens</th>
<th>Probabilistic modus ponens</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Conditional event)</td>
<td>(Material conditional)</td>
</tr>
<tr>
<td>If $A$, then $C$</td>
<td>$p(C</td>
</tr>
<tr>
<td>$A$</td>
<td>$p(A) = .50$</td>
</tr>
<tr>
<td>$C$</td>
<td>$0.45 \leq p(C) \leq 0.95$</td>
</tr>
<tr>
<td></td>
<td>$0.40 \leq p(C) \leq 0.90$</td>
</tr>
</tbody>
</table>

...where the consequence relation ("———") is deductive.
Consider a probability logical argument with $n$ premises:

Premise 1

\[ \ldots \]

Premise $n$

\[ \text{Conclusion} \]
Consider a probability logical argument with \( n \) premises:

\[
\begin{align*}
\text{Premise 1} & \implies \ldots \text{what the speaker says} \\
\ldots & \ldots \ldots \\
\text{Premise } n & \implies \ldots \text{what the speaker says} \\
\text{Conclusion} & \implies \ldots \text{what the listeners hears/infers}
\end{align*}
\]
Table of contents

Introduction
  Disciplines & interaction of normative and empirical work
  Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks

References
Sample paradoxes of the material conditional

(Paradox 1) \[
\begin{array}{c}
B \\
\hline
\text{If } A, \text{ then } B
\end{array}
\]

(Paradox 2) \[
\begin{array}{c}
\text{Not: } A \\
\hline
\text{If } A, \text{ then } B
\end{array}
\]
Sample paradoxes of the material conditional

(Paradox 1) \[ B \]
If \( A \), then \( B \)

(Paradox 1) \[ B \]
\[ A \Rightarrow B \]

(Paradox 2) \[ \neg A \]
If \( A \), then \( B \)

(Paradox 2) \[ \neg A \]
\[ A \Rightarrow B \]
Sample paradoxes of the material conditional

(Paradox 1) \[ P(B) = x \]
\[ x \leq P(A \supset B) \leq 1 \]

(Paradox 2) \[ P(\neg A) = x \]
\[ 1 - x \leq P(A \supset B) \leq 1 \]

probabilistically informative
Sample paradoxes of the material conditional

\[
\begin{align*}
\text{(Paradox 1)} & \quad P(B) = x \\
& \quad x \leq P(A \supset B) \leq 1
\end{align*}
\]

\[
\begin{align*}
\text{(Paradox 2)} & \quad P(\neg A) = x \\
& \quad 1 - x \leq P(A \supset B) \leq 1
\end{align*}
\]

probabilistically informative
Paradoxes of the material conditional, e.g.,

\[
\begin{align*}
(P\text{aradox 1}) & \quad P(B) = x \quad 0 \leq P(B|A) \leq 1 \\
(P\text{aradox 2}) & \quad P(\neg A) = x \quad 0 \leq P(B|A) \leq 1
\end{align*}
\]

probabilistically non-informative
Paradoxes of the material conditional, e.g.,

(Paradox 1) \[ P(B) = x \]

\[ 0 \leq P(B|A) \leq 1 \]

(Paradox 2) \[ P(\neg A) = x \]

\[ 0 \leq P(B|A) \leq 1 \]

probabilistically non-informative

This matches the data (Pfeifer & Kleiter, 2011).
Paradoxes of the material conditional, e.g.,

\[
\begin{align*}
\text{(Paradox 1)} \\
& \quad P(B) = x \\
& \quad 0 \leq P(B|A) \leq 1 \\
\end{align*}
\]

\[
\begin{align*}
\text{(Paradox 2)} \\
& \quad P(\neg A) = x \\
& \quad 0 \leq P(B|A) \leq 1 \\
\end{align*}
\]

probabilistically non-informative

This matches the data (Pfeifer & Kleiter, 2011).

**Paradox 1:** Special case covered in the coherence approach, but not covered in the standard approach to probability:

If \( P(B) = 1 \), then \( P(A \land B) = P(A) \).
Sample paradoxes of the material conditional (Pfeifer, 2014, *Studia Logica*)

Paradoxes of the material conditional, e.g.,

\[
\begin{align*}
\text{(Paradox 1)} & \quad P(B) = x \\
& \quad 0 \leq P(B|A) \leq 1
\end{align*}
\]

\[
\begin{align*}
\text{(Paradox 2)} & \quad P(\neg A) = x \\
& \quad 0 \leq P(B|A) \leq 1
\end{align*}
\]

probabilistically **non**-informative

This matches the data (Pfeifer & Kleiter, 2011).

**Paradox 1:** Special case covered in the coherence approach, but **not covered** in the standard approach to probability:

If \( P(B) = 1 \), then \( P(A \land B) = P(A) \). Thus,

\[
P(B|A) = \frac{P(A \land B)}{P(A)} = \frac{P(A)}{P(A)} = 1, \text{ if } P(A) > 0.
\]

From $\Pr(B) = 1$ and $A \land B \equiv \bot$ infer $\Pr(B | A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \land B \equiv \bot$ infer $\Pr(B | A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \supset B \equiv \top$ infer $\Pr(B | A) = 1$ is coherent.

From $\Pr(B) = 1$ and $A \land B \equiv \bot$ infer $\Pr(B | A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \supset B \equiv \top$ infer $\Pr(B | A) = 1$ is coherent.

From $\Pr(B) = x$ and $\Pr(A) = y$ infer

$$\max \left\{ 0, \frac{x + y - 1}{y} \right\} \leq \Pr(B | A) \leq \min \left\{ \frac{x}{y} , 1 \right\}$$

is coherent.
From $\text{Pr}(B) = 1$ and $A \land B \equiv \bot$ infer $\text{Pr}(B \mid A) = 0$ is coherent.

From $\text{Pr}(B) = 1$ and $A \supset B \equiv \top$ infer $\text{Pr}(B \mid A) = 1$ is coherent.

From $\text{Pr}(B) = x$ and $\text{Pr}(A) = y$ infer

$$\max\left\{0, \frac{x + y - 1}{y}\right\} \leq \text{Pr}(B \mid A) \leq \min\left\{\frac{x}{y}, 1\right\}$$

is coherent.

...a special case of the cautious monotonicity rule of System P

(Gilio, 2002).
Table of contents

Introduction
  Disciplines & interaction of normative and empirical work
  Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks

References
Probabilistic truth table task  
(Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

\[
\begin{align*}
P(A \land C) &= x_1 \\
P(A \land \neg C) &= x_2 \\
P(\neg A \land C) &= x_3 \\
P(\neg A \land \neg C) &= x_4 \\
\hline
P(\text{If } A, \text{ then } C) &= ?
\end{align*}
\]
Probabilistic truth table task  (Evans et al., 2003; Oberauer & Wilhelm, 2003)

\[
\begin{align*}
P(A \land C) &= x_1 \\
P(A \land \neg C) &= x_2 \\
P(\neg A \land C) &= x_3 \\
P(\neg A \land \neg C) &= x_4
\end{align*}
\]

\[
P(\text{If } A, \text{ then } C) = ?
\]

Conclusion candidates:

- \(P(A \land C) = x_1\)
- \(P(C|A) = x_1/(x_1 + x_2)\)
- \(P(A \supset C) = x_1 + x_3 + x_4\)
Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

\[
\begin{align*}
P(A \land C) & = x_1 = 0.25 \\
P(A \land \neg C) & = x_2 = 0.25 \\
P(\neg A \land C) & = x_3 = 0.25 \\
P(\neg A \land \neg C) & = x_4 = 0.25
\end{align*}
\]

\[
P(\text{If } A, \text{ then } C) = ?
\]

Conclusion candidates:

- \( P(A \land C) = x_1 \)
- \( P(C|A) = x_1/(x_1 + x_2) \)
- \( P(A \supset C) = x_1 + x_3 + x_4 \)
Proabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

\[
\begin{align*}
P(A \land C) &= x_1 = 0.25 \\
P(A \land \neg C) &= x_2 = 0.25 \\
P(\neg A \land C) &= x_3 = 0.25 \\
P(\neg A \land \neg C) &= x_4 = 0.25
\end{align*}
\]

Conclusion candidates:

- \( P(A \land C) = x_1 = 0.25 \)
- \( P(C|A) = x_1/(x_1 + x_2) = 0.50 \)
- \( P(A \supset C) = x_1 + x_3 + x_4 = 0.75 \)
Probabilistic truth table task  \( \text{Evans et al., 2003; Oberauer & Wilhelm, 2003} \)

\[
\begin{align*}
P(A \land C) &= x_1 \\
P(A \land \neg C) &= x_2 \\
P(\neg A \land C) &= x_3 \\
P(\neg A \land \neg C) &= x_4 \\
\hline
P(\text{If } A, \text{ then } C) &= ?
\end{align*}
\]

Main results:

- More than half of the responses are consistent with \( P(C|A) \)
- Many responses are consistent with \( P(A \land C) \)
Probabilistic truth table task \cite{Evans2003, Oberauer2003}

\[
\begin{align*}
P(A \land C) &= x_1 \\
P(A \land \neg C) &= x_2 \\
P(\neg A \land C) &= x_3 \\
P(\neg A \land \neg C) &= x_4 \\
P(\text{If } A, \text{ then } C) &= ?
\end{align*}
\]

Main results:

- More than half of the responses are consistent with \( P(C|A) \)
- Many responses are consistent with \( P(A \land C) \)
- **Generalized version:** Interpretation shifts to \( P(C|A) \) \cite{Fugard2011a, Mayerhofer2011a, JournalOfExperimentalPsychologyLMC}
Probabilistic truth table task  (Evans et al., 2003; Oberauer & Wilhelm, 2003)

\[
\begin{align*}
P(A \land C) &= x_1 \\
P(A \land \neg C) &= x_2 \\
P(\neg A \land C) &= x_3 \\
P(\neg A \land \neg C) &= x_4 \\
\hline
P(\text{If } A \text{, then } C) &= ?
\end{align*}
\]

Main results:

- More than half of the responses are consistent with \( P(C|A) \)
- Many responses are consistent with \( P(A \land C) \)
- Generalized version: Interpretation shifts to \( P(C|A) \)  (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011a, *Journal of Experimental Psychology: LMC*)

Key feature:

- Reasoning under complete probabilistic knowledge
Experiment

Motivation

▸ probabilistic truth table task with incomplete probabilistic knowledge
▸ Is the conditional event interpretation still dominant?
▸ Are there shifts of interpretation?
Example: Task 5  (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black or white*) and a shape (*circle, triangle, or square*). Question marks indicate covered sides.

[Diagram of die sides with black square, white triangle, and question marks]
Illustrated here are all sides of a six-sided die. The sides have two properties: a color (black or white) and a shape (circle, triangle, or square). Question marks indicate covered sides.

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.
Example: Task 5  (Pfeifer, 2013a, Thinking & Reasoning)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (black or white) and a shape (circle, triangle, or square). Question marks indicate covered sides.

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

**Question:** How sure can you be that the following sentence holds?

If the side facing up shows white, then the side shows a square.
Example: Task 5  (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.

![Die Sides](image)

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

**Question:** How sure can you be that the following sentence holds?

**If** the side facing up shows *white*, **then** the side shows a *square*.

**Answer:**

<table>
<thead>
<tr>
<th></th>
<th>at least</th>
<th></th>
<th>at most</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Boxes" /></td>
<td><img src="image" alt="Boxes" /></td>
<td><img src="image" alt="Boxes" /></td>
<td><img src="image" alt="Boxes" /></td>
</tr>
</tbody>
</table>

(please tick the appropriate boxes)
Illustrated here are all sides of a six-sided die. The sides have two properties: a color (black or white) and a shape (circle, triangle, or square). Question marks indicate covered sides.

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

**Question:** How sure can you be that the following sentence holds?

**If** the side facing up shows white, **then** the side shows a square.

**Answer:** Cond. event: at least 1 out of 5 and at most 3 out of 5

<table>
<thead>
<tr>
<th>at least</th>
<th>at most</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="boxes" /></td>
<td><img src="image2.png" alt="boxes" /></td>
</tr>
</tbody>
</table>

(please tick the appropriate boxes)
Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (black or white) and a shape (circle, triangle, or square). Question marks indicate covered sides.

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

**Question:** How sure can you be that the following sentence holds?

**If** the side facing up shows white, **then** the side shows a square.

**Answer:** Conjunction: at least 1 out of 6 and at most 3 out of 6

(please tick the appropriate boxes)
Example: Task 5  (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.

![Die Sides](image)

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

**Question:** How sure can you be that the following sentence holds?

> If the side facing up shows *white*, then the side shows a *square*.

**Answer:** Mat. cond.: *at least* 2 out of 6 and *at most* 4 out of 6

<table>
<thead>
<tr>
<th>at least</th>
<th>at most</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Options" /></td>
<td><img src="image" alt="Options" /></td>
</tr>
</tbody>
</table>

(out of 0 1 2 3 4 5 6)

(please tick the appropriate boxes)
Experiment (Pfeifer, 2013a, Thinking & Reasoning)

Set-up

- 20 tasks, three “warming-up tasks”
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation
Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

**Set-up**
- 20 tasks, three “warming-up tasks”
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

**Sample**
- 20 Cambridge University students
- 10 female, 10 male
- between 18 and 27 years old (mean: 21.65)
- no students of mathematics, philosophy, computer science, or psychology
Experiment (Pfeifer, 2013a, Thinking & Reasoning)

Set-up

- 20 tasks, three “warming-up tasks”
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional
Experiment (Pfeifer, 2013a, Thinking & Reasoning)

Set-up

- 20 tasks, three “warming-up tasks”
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional

- Shift of interpretation
  - First three tasks: 38.3% consistent with conditional event
  - Last three tasks: 83.3% consistent with conditional event
  - Strong correlation between conditional event frequency and item position ($r(15) = 0.71, p < 0.005$)
Increase of cond. event resp. \( (n_1 = 20) \) (Pfeifer, 2013a, *Thinking & Reasoning*)
Further observations

- Conditional probability responses are also clearly dominant in PTT tasks using counterfactuals (Pfeifer & Stöckle-Schobel, 2015)
Further observations

- Conditional probability responses are also clearly dominant in PTT tasks using **counterfactuals** (Pfeifer & Stöckle-Schobel, 2015) and ...
- ... in **causal scenarios** (Pfeifer & Stöckle-Schobel, 2015).
Further observations

- Conditional probability responses are also clearly dominant in PTT tasks using counterfactuals (Pfeifer & Stöckle-Schobel, 2015) and...

- ... in causal scenarios (Pfeifer & Stöckle-Schobel, 2015).

- Apparent pragmatic/relevance effect when “packed” (e.g., “If the card shows a 2, then the card shows an even number”) and “unpacked” (“If the card shows a 2, then the card shows a 2 or a 4”) conditionals are compared
Further observations

- Conditional probability responses are also clearly dominant in PTT tasks using counterfactuals (Pfeifer & Stöckle-Schobel, 2015) and . . .
- . . . in causal scenarios (Pfeifer & Stöckle-Schobel, 2015).
- Apparent pragmatic/relevance effect when “packed” (e.g., “If the card shows a 2, then the card shows an even number”) and “unpacked” (“If the card shows a 2, then the card shows a 2 or a 4”) conditionals are compared:
  Most people judge (correctly) $p(\text{even}|x = 2) = 1$
Further observations

- Conditional probability responses are also clearly dominant in PTT tasks using counterfactuals (Pfeifer & Stöckle-Schobel, 2015) and . . .
- . . . in causal scenarios (Pfeifer & Stöckle-Schobel, 2015).
- Apparent pragmatic/relevance effect when “packed” (e.g., “If the card shows a 2, then the card shows an even number”) and “unpacked” (“If the card shows a 2, then the card shows a 2 or a 4”) conditionals are compared: Most people judge (correctly) \( p(\text{even}|x = 2) = 1 \)
  but (incorrectly) \( p(x = 2 \lor x = 4|x = 2) = 0 \)
  (Fugard, Pfeifer, & Mayerhofer, 2011).
Table of contents

Introduction
  Disciplines & interaction of normative and empirical work
  Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks

References
The Tweety problem
The Tweety problem
System P: Rationality postulates for nonmonotonic reasoning  (Kraus, Lehmann, & Magidor, 1990)

Reflexivity (axiom):  $\alpha \nvdash \neg \alpha$

Left logical equivalence:
from $\models \alpha \equiv \beta$ and $\alpha \nvdash \gamma$ infer $\beta \nvdash \gamma$

Right weakening:
from $\models \alpha \supset \beta$ and $\gamma \nvdash \alpha$ infer $\gamma \nvdash \beta$

Or:
from $\alpha \nvdash \gamma$ and $\beta \nvdash \gamma$ infer $\alpha \lor \beta \nvdash \gamma$

Cut:
from $\alpha \land \beta \nvdash \gamma$ and $\alpha \nvdash \beta$ infer $\alpha \nvdash \gamma$

Cautious monotonicity:
from $\alpha \nvdash \beta$ and $\alpha \nvdash \gamma$ infer $\alpha \land \beta \nvdash \gamma$

And (derived rule): from $\alpha \nvdash \beta$ and $\alpha \nvdash \gamma$ infer $\alpha \nvdash \beta \land \gamma$
System P: Rationality postulates for nonmonotonic reasoning\textsuperscript{(Kraus et al., 1990)}

Reflexivity (axiom): \( \alpha \not\vdash \alpha \)

Left logical equivalence:
\[
\text{from } \vdash \alpha \equiv \beta \text{ and } \alpha \not\vdash \gamma \text{ infer } \beta \not\vdash \gamma
\]

Right weakening:
\[
\text{from } \vdash \alpha \supset \beta \text{ and } \gamma \not\vdash \alpha \text{ infer } \gamma \not\vdash \beta
\]

Or:
\[
\text{from } \alpha \not\vdash \gamma \text{ and } \beta \not\vdash \gamma \text{ infer } \alpha \lor \beta \not\vdash \gamma
\]

Cut:
\[
\text{from } \alpha \land \beta \not\vdash \gamma \text{ and } \alpha \not\vdash \beta \text{ infer } \alpha \not\vdash \gamma
\]

Cautious monotonicity:
\[
\text{from } \alpha \not\vdash \beta \text{ and } \alpha \not\vdash \gamma \text{ infer } \alpha \land \beta \not\vdash \gamma
\]

And (derived rule): from \( \alpha \not\vdash \beta \) and \( \alpha \not\vdash \gamma \) infer \( \alpha \not\vdash \beta \land \gamma \)

\( \alpha \not\vdash \beta \) is read as \( \text{If } \alpha, \text{ normally } \beta \)
<table>
<thead>
<tr>
<th>Name</th>
<th>Probability logical version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left logical equivalence</td>
<td>( E_1 \equiv E_2 ), ( P(E_3</td>
</tr>
<tr>
<td>Right weakening</td>
<td>( P(E_1</td>
</tr>
<tr>
<td>Cut</td>
<td>( P(E_2</td>
</tr>
<tr>
<td>And</td>
<td>( P(E_2</td>
</tr>
<tr>
<td>Cautious monotonicity</td>
<td>( P(E_3</td>
</tr>
<tr>
<td>Or</td>
<td>( P(E_3</td>
</tr>
<tr>
<td>Transitivity</td>
<td>( P(E_2</td>
</tr>
<tr>
<td>Contraposition</td>
<td>( P(E_2</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>( P(E_3</td>
</tr>
</tbody>
</table>
## Probabilistic version of System P

(Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

<table>
<thead>
<tr>
<th>Name</th>
<th>Probability logical version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left logical equivalence</td>
<td>$\vdash (E_1 \equiv E_2), P(E_3</td>
</tr>
<tr>
<td>Right weakening</td>
<td>$P(E_1</td>
</tr>
<tr>
<td>Cut</td>
<td>$P(E_2</td>
</tr>
<tr>
<td></td>
<td>$\therefore P(E_2</td>
</tr>
<tr>
<td>And</td>
<td>$P(E_2</td>
</tr>
<tr>
<td></td>
<td>$\therefore P(E_2 \land E_3</td>
</tr>
<tr>
<td>Cautious monotonicity</td>
<td>$P(E_2</td>
</tr>
<tr>
<td></td>
<td>$\therefore P(E_3</td>
</tr>
<tr>
<td>Or</td>
<td>$P(E_3</td>
</tr>
<tr>
<td></td>
<td>$\therefore P(E_3</td>
</tr>
<tr>
<td>Transitivity</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Contraposition</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>$P(E_3</td>
</tr>
</tbody>
</table>

... where $\therefore$ is deductive
Probabilistic version of System P  
(Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

<table>
<thead>
<tr>
<th>Name</th>
<th>Probability logical version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left logical equivalence</td>
<td>$\models (E_1 \equiv E_2), P(E_3</td>
</tr>
<tr>
<td>Right weakening</td>
<td>$P(E_1</td>
</tr>
<tr>
<td>Cut</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>And</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Cautious monotonicity</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Or</td>
<td>$P(E_3</td>
</tr>
<tr>
<td>Transitivity</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Contraposition</td>
<td>$P(E_2</td>
</tr>
<tr>
<td>Monotonicity</td>
<td>$P(E_3</td>
</tr>
</tbody>
</table>

... where $\therefore$ is deductive

... probabilistically non-informative
The Tweety problem  (Pfeifer, 2012b)

\( \mathcal{P}_1 \quad P[\text{Fly}(x)|\text{Bird}(x)] = .95. \)  
\( \quad (Birds \ can \ normally \ fly.) \)

\( \mathcal{P}_2 \quad \text{Bird}(Tweety). \)  
\( \quad (Tweety \ is \ a \ bird.) \)

\( \mathcal{C}_1 \quad P[\text{Fly}(Tweety)] = .95. \)  
\( \quad (Tweety \ can \ normally \ fly.) \)
The Tweety problem  (Pfeifer, 2012b)

\[ \begin{align*}
\mathcal{P}_1 & \quad P[\text{Fly}(x) \mid \text{Bird}(x)] = .95. \quad (\text{Birds can normally fly.}) \\
\mathcal{P}_2 & \quad \text{Bird(Tweety).} \quad (\text{Tweety is a bird.}) \\
\mathcal{C}_1 & \quad \frac{P[\text{Fly(Tweety)}]}{P[\text{Bird(Tweety)}]} = .95. \quad (\text{Tweety can normally fly.}) \\
\mathcal{P}_3 & \quad \text{Penguin(Tweety).} \quad (\text{Tweety is a penguin.}) \\
\mathcal{P}_4 & \quad P[\text{Fly}(x) \mid \text{Penguin}(x)] = .01. \quad (\text{Penguins normally can't fly.}) \\
\mathcal{P}_5 & \quad P[\text{Bird}(x) \mid \text{Penguin}(x)] = .99. \quad (\text{Penguins are normally birds.}) \\
\mathcal{C}_2 & \quad P[\text{Fly(Tweety)} \mid \text{Bird(Tweety)} \land \text{Penguin(Tweety)}] \in [0, .01]. \quad (\text{If Tweety is a bird and a penguin, normally Tweety can't fly.})
\end{align*} \]
### The Tweety problem (Pfeifer, 2012b)

<table>
<thead>
<tr>
<th>前提</th>
<th>表达</th>
<th>推论</th>
<th>表达</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$P[\text{Fly}(x)</td>
<td>\text{Bird}(x)] = .95.$</td>
<td>$\text{C}_1$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$\text{Bird}(\text{Tweety})$.</td>
<td></td>
<td>$(\text{Tweety is a bird.})$</td>
</tr>
<tr>
<td>$\text{C}_1$</td>
<td>$P[\text{Fly}(\text{Tweety})] = .95.$</td>
<td></td>
<td>$(\text{Tweety can normally fly.})$</td>
</tr>
<tr>
<td>$P_3$</td>
<td>$\text{Penguin}(\text{Tweety})$.</td>
<td></td>
<td>$(\text{Tweety is a penguin.})$</td>
</tr>
<tr>
<td>$P_4$</td>
<td>$P[\text{Fly}(x)</td>
<td>\text{Penguin}(x)] = .01.$</td>
<td></td>
</tr>
<tr>
<td>$P_5$</td>
<td>$P[\text{Bird}(x)</td>
<td>\text{Penguin}(x)] = .99.$</td>
<td></td>
</tr>
<tr>
<td>$\text{C}_2$</td>
<td>$P[\text{Fly}(\text{Tweety})</td>
<td>\text{Bird}(\text{Tweety}) \land \text{Penguin}(\text{Tweety})] \in [0, .01]$.</td>
<td></td>
</tr>
</tbody>
</table>

The probabilistic modus ponens justifies $\text{C}_1$ and cautious monotonicity justifies $\text{C}_2$. 
The Tweety problem  

(Pfeifer, 2012b)

\[ P[\text{Fly}(x)|\text{Bird}(x)] = .95. \]  
\[ (\text{Birds can normally fly.}) \]

\[ \text{Bird}(\text{Tweety}). \]  
\[ (\text{Tweety is a bird.}) \]

\[ P[\text{Fly}(\text{Tweety})] = .95. \]  
\[ (\text{Tweety can normally fly.}) \]

\[ \text{Penguin}(\text{Tweety}). \]  
\[ (\text{Tweety is a penguin.}) \]

\[ P[\text{Fly}(x)|\text{Penguin}(x)] = .01. \]  
\[ (\text{Penguins normally can't fly.}) \]

\[ P[\text{Bird}(x)|\text{Penguin}(x)] = .99. \]  
\[ (\text{Penguins are normally birds.}) \]

\[ P[\text{Fly}(\text{Tweety}) | \text{Bird}(\text{Tweety}) \land \text{Penguin}(\text{Tweety})] \in [0, .01]. \]  
\[ (\text{If Tweety is a bird and a penguin, normally Tweety can't fly.}) \]

The probabilistic modus ponens justifies \( \mathcal{C}_1 \) and cautious monotonicity justifies \( \mathcal{C}_2 \).
Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitivity</td>
<td>$A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$</td>
</tr>
</tbody>
</table>
### Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitivity</td>
<td>$A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$</td>
</tr>
<tr>
<td></td>
<td>$P(B</td>
</tr>
</tbody>
</table>
## Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
</table>
| Transitivity       | $A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$  
                    | $P(B|A) = x, P(C|B) = y \therefore P(C|A) \in [0,1]$                                                                                     |
| Right weakening    | $P(B|A) = x, \models (B \supset C) \therefore P(C|A) \in [x,1]$                                                                         |
## Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitivity</td>
<td>$A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$</td>
</tr>
<tr>
<td></td>
<td>$P(B</td>
</tr>
<tr>
<td>Right weakening</td>
<td>$P(B</td>
</tr>
<tr>
<td>Cut</td>
<td>$P(B</td>
</tr>
<tr>
<td></td>
<td>$\therefore P(C</td>
</tr>
</tbody>
</table>
Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
</table>
| Transitivity          | $A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$  
  $P(B|A) = x, P(C|B) = y \therefore P(C|A) \in [0, 1]$ |
| Right weakening       | $P(B|A) = x, \vdash (B \supset C) \therefore P(C|A) \in [x, 1]$ |
| Cut                  | $P(B|A) = x, P(C|A \land B) = x, \therefore P(C|A) \in [xy, 1 - x + xy]$ |

- **Experimental result:** Right weakening is endorsed by almost all participants (Pfeifer & Kleiter, 2006b, 2010)
Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
</table>
| Transitivity     | $A \rightarrow B, B \rightarrow C$, therefore $A \rightarrow C$  
                    $P(B|A) = x, P(C|B) = y \therefore P(C|A) \in [0, 1]$ |
| Right weakening  | $P(B|A) = x, \models (B \supset C) \therefore P(C|A) \in [x, 1]$ |
| Cut              | $P(B|A) = x, P(C|A \land B) = x$, \therefore $P(C|A) \in [xy, 1 - x + xy]$ |

- **Experimental result:** Right weakening is endorsed by almost all participants (Pfeifer & Kleiter, 2006b, 2010)
- **Observation:** Deleting “A” in Cut yields Modus Ponens.
### Selected forms of transitivity & empirical evidence

<table>
<thead>
<tr>
<th>Name</th>
<th>Formalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitivity</td>
<td>( A \rightarrow B, B \rightarrow C, \text{ therefore } A \rightarrow C )</td>
</tr>
<tr>
<td></td>
<td>( P(B</td>
</tr>
<tr>
<td>Right weakening</td>
<td>( P(B</td>
</tr>
<tr>
<td>Cut</td>
<td>( P(B</td>
</tr>
<tr>
<td></td>
<td>( \therefore \quad P(C</td>
</tr>
</tbody>
</table>

- **Experimental result**: Right weakening is endorsed by almost all participants (Pfeifer & Kleiter, 2006b, 2010)

- **Observation**: Deleting “A” in Cut yields Modus Ponens.

- **Experimental result**: Non-probabilistic tasks: endorsement rate of 89–100% (Evans et al., 1993); probabilistic tasks: 63%–100% coherent responses (Pfeifer & Kleiter, 2007)
The Transitivity Task

Please imagine the following situation:

Exactly 99% of the cars on a big parking lot are blue. Exactly 63% of blue cars have grey wheel rims.
Please imagine the following situation:

Exactly 99% of the cars on a big parking lot are blue.
Exactly 63% of blue cars have grey wheel rims.

Imagine all the cars that are on the big parking lot. How many of these cars have grey wheel rims?
Please imagine the following situation:

Exactly 99% of the cars on a **big parking lot** are **blue**.
Exactly 63% of **blue cars** that are on the **big parking lot**
  have **grey wheel rims**.

Imagine all the cars that are on the **big parking lot**. How many of
these cars have **grey wheel rims**?

(Adams, 1975; Bennett, 2003)
Results: Transitivity... “as Cut”  
(Pfeifer & Kleiter, 2006b)

coherent interval

averaged interval response frequencies, 14 tasks, $n = 20$
Table of contents

Introduction
  Disciplines & interaction of normative and empirical work
  Mental probability logic

Paradoxes of the material conditional

Probabilistic truth tables

Nonmonotonic reasoning

Concluding remarks
References
Concluding remarks

- Coherence-based probability logic as a rationality framework for a “probabilistic experimental pragmatics”
Concluding remarks

- **Coherence-based** probability logic as a rationality framework for a “probabilistic experimental pragmatics”

- Premises represent what the speaker says and conclusions represent what the hearer infers
Concluding remarks

- Coherence-based probability logic as a rationality framework for a “probabilistic experimental pragmatics”
- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret conditionals by conditional probabilities:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
Concluding remarks

- Coherence-based probability logic as a rationality framework for a “probabilistic experimental pragmatics”
- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret conditionals by conditional probabilities:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
- Most people draw coherent inferences.
Concluding remarks

- **Coherence-based** probability logic as a rationality framework for a “probabilistic experimental pragmatics”
- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret **conditionals** by **conditional probabilities**:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
- **Most** people draw **coherent** inferences. Specifically:
  - **Conditional probability** responses are consistently the **dominant** responses in the **paradox** tasks, the **probabilistic truth table** tasks, and the **nonmonotonic reasoning** tasks
Concluding remarks

- Coherence-based probability logic as a rationality framework for a “probabilistic experimental pragmatics”
- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret conditionals by conditional probabilities:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
- Most people draw coherent inferences. Specifically:
  - Conditional probability responses are consistently the dominant responses in the paradox tasks, the probabilistic truth table tasks, and the nonmonotonic reasoning tasks
- True interaction of formal and empirical work: opens interdisciplinary collaborations
Concluding remarks

- Coherence-based probability logic as a rationality framework for a “probabilistic experimental pragmatics”
- Premises represent what the speaker says and conclusions represent what the hearer infers
- Interpret conditionals by conditional probabilities:
  - to avoid paradoxes
  - to withdraw conclusions in the light of new evidence
- Most people draw coherent inferences. Specifically:
  - Conditional probability responses are consistently the dominant responses in the paradox tasks, the probabilistic truth table tasks, and the nonmonotonic reasoning tasks
- True interaction of formal and empirical work: opens interdisciplinary collaborations
- Long term goal: Theory of uncertain inference which is normatively and descriptively adequate

Papers available at: www.pfeifer-research.de
Contact: <niki.pfeifer@lmu.de>
Appendix
Properties of arguments

An argument is a pair consisting of a premise set and a conclusion.

- An argument is **logically valid** if and only if it is impossible that all premises are true and the conclusion is false.
Properties of arguments

An argument is a pair consisting of a premise set and a conclusion.

- An argument is **logically valid** if and only if it is impossible that all premises are true and the conclusion is false.
- An argument is **p-valid** if and only if the uncertainty of the conclusion of a valid inference cannot exceed the sum of the uncertainties of its premises (where “uncertainty of $X$” is defined by $1 - P(X)$) (Adams, 1975).
Properties of arguments

An argument is a pair consisting of a premise set and a conclusion.

- An argument is **logically valid** if and only if it is impossible that all premises are true and the conclusion is false.
- An argument is **$p$-valid** if and only if the uncertainty of the conclusion of a valid inference cannot exceed the sum of the uncertainties of its premises (where “uncertainty of $X$” is defined by $1 - P(X)$) (Adams, 1975).
- An argument is **probabilistically informative** if and only if it is possible that the premise probabilities constrain the conclusion probability. I.e., if the coherent probability interval of its conclusion is not necessarily equal to the unit interval $[0, 1]$ (Pfeifer & Kleiter, 2006a).
Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)
Logically valid—probabilistically informative (Pfeifer & Kleiter (2009). Journal of Applied Logic. Figure 1)
Logically valid—probabilistically informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)
Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)

![Diagram of logical validity and probabilistic informativeness](image-url)
Logically valid–probabilistically informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)
Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)
Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)
Mental probability logic II

- uncertain argument forms
  - conditional syllogisms (Pfeifer & Kleiter, 2007, 2009)
  - monotonic and non-monotonic arguments (Pfeifer & Kleiter, 2005a, 2010)
  - nested conditionals (Gilio, Over, Pfeifer, & Sanfilippo, in press)
Mental probability logic II

- uncertain argument forms
  - conditional syllogisms (Pfeifer & Kleiter, 2007, 2009)
  - monotonic and non-monotonic arguments (Pfeifer & Kleiter, 2005a, 2010)
  - nested conditionals (Gilio, Over, Pfeifer, & Sanfilippo, in press)
- argumentation
  - strength of argument forms (Pfeifer & Kleiter, 2006a)
  - fallacies (Pfeifer, 2008)
Mental probability logic II

▸ uncertain argument forms
  ▸ conditional syllogisms (Pfeifer & Kleiter, 2007, 2009)
  ▸ monotonic and non-monotonic arguments (Pfeifer & Kleiter, 2005a, 2010)
  ▸ nested conditionals (Gilio, Over, Pfeifer, & Sanfilippo, in press)

▸ argumentation
  ▸ strength of argument forms (Pfeifer & Kleiter, 2006a)
    and strength of concrete arguments (Pfeifer, 2007, 2013b)
  ▸ fallacies (Pfeifer, 2008)

▸ conditional reasoning
  ▸ probabilistic truth table task
    ▸ shifts of interpretation (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011)
    ▸ incomplete probabilistic knowledge (Pfeifer, 2013a)
  ▸ Aristotle’s thesis (Pfeifer, 2012a)
  ▸ paradoxes of the material conditional (Pfeifer & Kleiter, 2011; Pfeifer, 2014)
Mental probability logic II

- uncertain **argument forms**
  - conditional syllogisms (Pfeifer & Kleiter, 2007, 2009)
  - monotonic and non-monotonic arguments (Pfeifer & Kleiter, 2005a, 2010)
  - nested conditionals (Gilio, Over, Pfeifer, & Sanfilippo, in press)
- **argumentation**
  - strength of argument forms (Pfeifer & Kleiter, 2006a)
  - fallacies (Pfeifer, 2008)
- **conditional reasoning**
  - probabilistic truth table task
    - shifts of interpretation (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011)
    - incomplete probabilistic knowledge (Pfeifer, 2013a)
  - Aristotle’s thesis (Pfeifer, 2012a)
  - paradoxes of the material conditional (Pfeifer & Kleiter, 2011; Pfeifer, 2014)
- **quantification**
  - frequency-based semantics (Pfeifer, 2006a)
  - coh.-based prob. semantics (Pfeifer, Sanfilippo, & Gilio, 2016)
  - square of opposition (Pfeifer & Sanfilippo, in press)
- Relation to **formal epistemology** (Pfeifer, 2012b; Pfeifer & Douven, 2014)
Mental probability logic II

- uncertain argument forms
  - conditional syllogisms (Pfeifer & Kleiter, 2007, 2009)
  - monotonic and non-monotonic arguments (Pfeifer & Kleiter, 2005a, 2010)
  - nested conditionals (Gilio, Over, Pfeifer, & Sanfilippo, in press)
- argumentation
  - strength of argument forms (Pfeifer & Kleiter, 2006a)
    and strength of concrete arguments (Pfeifer, 2007, 2013b)
  - fallacies (Pfeifer, 2008)
- conditional reasoning
  - probabilistic truth table task
    - shifts of interpretation (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011)
    - incomplete probabilistic knowledge (Pfeifer, 2013a)
  - Aristotle’s thesis (Pfeifer, 2012a)
  - paradoxes of the material conditional (Pfeifer & Kleiter, 2011; Pfeifer, 2014)
- quantification
  - frequency-based semantics (Pfeifer, 2006a)
  - coh.-based prob. semantics (Pfeifer, Sanfilippo, & Gilio, 2016)
  - square of opposition (Pfeifer & Sanfilippo, in press)
- Relation to formal epistemology (Pfeifer, 2012b; Pfeifer & Douven, 2014)
Example 1: (Cautious) monotonicity

- In logic
  from $A \supset B$ infer $(A \land C) \supset B$

- In probability logic
  from $P(B|A) = x$ infer $0 \leq P(B|A \land C) \leq 1$
Example 1: (Cautious) monotonicity

- In logic
  from $A \supset B$ infer $(A \land C) \supset B$

- In probability logic
  from $P(B|A) = x$ infer $0 \leq P(B|A \land C) \leq 1$
  But: from $P(A \supset B) = x$ infer $x \leq P((A \land C) \supset B) \leq 1$
Example 1: (Cautious) monotonicity

- In logic
  from $A \supset B$ infer $(A \land C) \supset B$

- In probability logic
  from $P(B|A) = x$ infer $0 \leq P(B|A \land C) \leq 1$
  But: from $P(A \supset B) = x$ infer $x \leq P((A \land C) \supset B) \leq 1$

- Cautious monotonicity (Gilio, 2002)
  from $P(B|A) = x$ and $P(C|A) = y$
  infer $\max(0, (x + y - 1)/x) \leq P(C|A \land B) \leq \min(y/x, 1)$
Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

*exactly 72% wear a black suit.*
Example task: Monotonicity  (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

*exactly 72% wear a black suit.*

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit, given they are at this prom and wear glasses?
Example task: Cautious monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

- exactly 72% wear a black suit.
- exactly 63% wear glasses.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit, given they are at this prom and wear glasses?
Results – Monotonicity  (Example Task 1; Pfeifer and Kleiter (2003))

lower  bound responses

upper  bound responses

\((n_1 = 20)\)
Results – Cautious monotonicity (Example Task 1; Pfeifer and Kleiter (2003))

lower bound responses

upper bound responses

\((n_2 = 19)\)
Example 2: Contraposition

- In logic
  - from $A \supset B$ infer $\neg B \supset \neg A$
  - from $\neg B \supset \neg A$ infer $A \supset B$
Example 2: Contraposition

- **In logic**
  - from $A \supset B$ infer $\neg B \supset \neg A$
  - from $\neg B \supset \neg A$ infer $A \supset B$

- **In probability logic**
  - from $P(B|A) = x$ infer $0 \leq P(\neg A | \neg B) \leq 1$
  - from $P(\neg A | \neg B) = x$ infer $0 \leq P(B|A) \leq 1$
Example 2: Contraposition

- In logic
  - from $A \supset B$ infer $\neg B \supset \neg A$
  - from $\neg B \supset \neg A$ infer $A \supset B$

- In probability logic
  - from $P(B|A) = x$ infer $0 \leq P(\neg A|\neg B) \leq 1$
  - from $P(\neg A|\neg B) = x$ infer $0 \leq P(B|A) \leq 1$

- But

\[ P(A \supset B) = P(\neg B \supset \neg A) \]
Results Contraposition \((n_1 = 40, n_2 = 40; \text{Pfeifer and Kleiter (2006b)})\)


References VII


