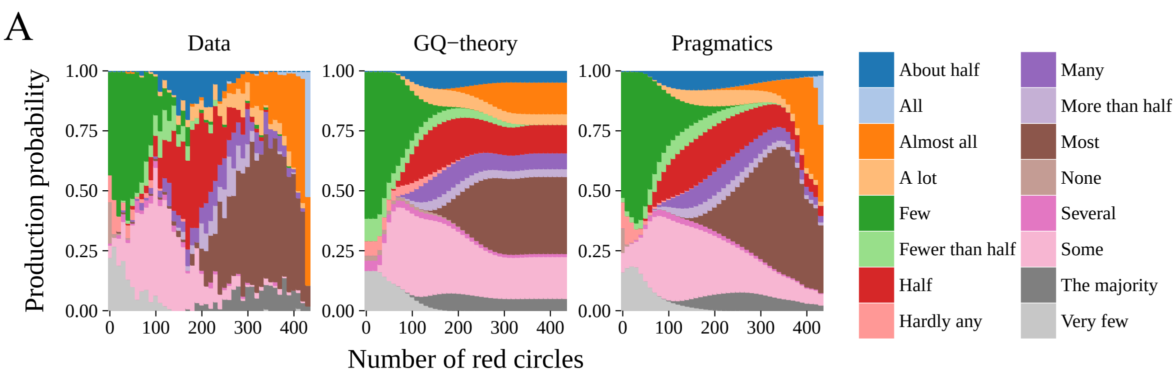
Speaking of quantifiers: modelling the production of quantity words

According to the *theory of generalised quantifiers* (GQ-theory), quantity words (e.g., ‘some’, ‘most’, ‘all’) express a relation between the set denoted by the subject S and the set denoted by the predicate P—specifically, between the intersection set size $S \cap P$ and the total set size S (e.g., Barwise & Cooper, 1981). This set-theoretic approach has proven itself immensely successful in explaining a range of observations (e.g., Katsos et al., 2016). At the same time, however, GQ-theory also has an important limitation: as it stands, it fails to offer a compelling account for the way speakers use quantity words in actual speech situations. To illustrate, we asked 600 participants to each describe the number of red circles in displays with 432 circles, each of which was either red or black. The left panel of Fig. A shows the binned production probabilities of the most frequently mentioned quantity words.



The data show that speakers associate quantity words with ranges of intersection set sizes that are narrower (e.g., ‘some’) or broader (e.g., ‘half’) than their set-theoretic definitions would suggest. We refer to the ranges that speakers associate with quantity words as their *focal ranges*. These focal ranges have a gradient structure, which seems to speak against the GQ-theoretic assumption that quantified sentences are always true or false simpliciter.

One might conclude from these data that the set-theoretic definitions postulated by GQ-theory—although perhaps relevant in the minds of logicians—have no role in actual linguistic communication, and that GQ-theory should be supplanted by, or at least enriched with, another theory of quantification (e.g., Zadeh, 1983). We show that this conclusion is premature, and argue that the discrepancy between GQ-theory and the production data can be bridged once we consider the pragmatics of communication; specifically, once we assume that speakers are rational, i.e., choose those messages that are most likely to receive the intended interpretation (e.g., Frank & Goodman, 2015).

To show how GQ-theory can be salvaged in light of the presence of focal ranges, we formulate a computational model of the production of quantity words. The model assumes that the speaker S attempts to communicate an intersection set size $t \in T = \{t_0, \ldots, t_{432}\}$ by producing a message $m \in M = \{m_{\text{‘about half’}}, \ldots, m_{\text{‘very few’}}\}$. We assume that the probability with which S chooses m to communicate t depends on three factors: the salience of m, the perceived intersection set size t', and the meaning of m.

As a measure of salience, we consider the overall frequency n of each quantity word in our production study: $P_M(m) \propto n_m$. It seems likely that the perceived intersection set size t' might differ from the actual intersection set size t. Thus, we define $P_C(t' \mid t, w)$ as the probability of confusing t with t'. The probabilities are defined by analogy to confusion probabilities of the Approximate Number System, i.e., the human cognitive system for estimating numerosity (e.g., Dehaene, 2001), with w representing Weber's fraction, which determines the overall accuracy of these estimates.

To capture the meaning of quantity words, we introduce a function B_θ that maps pairs of m and t to the truth value of m in t. According to GQ-theory, many quantity words denote either a lower or upper bound on the intersection set size. Based on a pretest in which 60 participants judged inference patterns from sets to supersets (e.g., does 'Some guests drank beer' imply 'Some guests drank alcohol'?) or from sets to subsets (e.g., does 'Some guests drank alcohol' imply 'Some guests drank beer'?), we determined that 'about half', 'all', 'almost all', 'a lot', 'half', 'many', 'more than half', 'most', 'several', and 'the majority' denote a lower bound, and the other quantity words denote an upper bound.

Combining these three ingredients—i.e., salience, confusability, and meaning—we can define probabilistic production behaviour of a literal speaker S_{Lit} who chooses messages purely based on their meanings, as follows:

$$P_{S_{Lit}}(m \mid t; \theta, w) \propto \sum_{t' \in T} P_C(t' \mid t; w)\, P_M(m)\, B_\theta(m, t')$$

A pragmatic speaker seeks to optimise the probability that the hearer infers the correct t. Hence, in order to operationalise the pragmatic speaker, we first need to determine how a hearer would behave. Given a message, a naive hearer H would infer an intersection set size with a probability proportional to its truth value, as determined by B_θ: $P_H(t \mid m) \propto B_\theta(m, t)$. The probability that a pragmatic speaker produces a message is determined by the probability that H arrives at the correct interpretation. The parameter λ regulates how likely it is that the pragmatic speaker produces the optimal quantity word. Higher values of λ indicate that the speaker is more likely to behave optimally: $P_S(m \mid t, \lambda) \propto P_H(t \mid m)^\lambda$. The following definition represents the behaviour of the pragmatic speaker S_{Prg}:

$$P_{S_{Prg}}(m \mid t; \theta, w, \lambda) \propto \sum_{t' \in T} P_C(t' \mid t; w)\, P_M(m)\, P_S(m \mid t'; \theta, \lambda)$$

To determine which speaker best accounts for the production data, we optimised the values of θ and w for S_{Lit}, and of θ, w, and λ for S_{Prg}. The middle and right panels of Fig. A show the binned optimal predictions for S_{Lit} (GQ-theory) and S_{Prg} (Pragmatics).

The semantic model S_{Lit} is unable to account for the presence of focal ranges, particularly for higher intersection set sizes. The correlation between the behaviour of S_{Lit} and the production data is .809. The pragmatic model S_{Prg} is substantially better in accounting for the focal ranges. Correspondingly, the overall correlation between the behaviour of S_{Prg} and the production data increased to .924. We thus show that the presence and location of focal ranges can be reconciled with the conservative GQ-theory if it is embedded in a probabilistic model of pragmatic reasoning that also incorporates other cognitive factors, such as imprecision in the perception and representation of quantities.

References: **[1]** Barwise, J. & Cooper, R. (1981). *L&P*, *4* (2), 159–219. **[2]** Dehaene, S. (1997). *The number sense: how the mind creates mathematics.* **[3]** Frank, M. & Goodman, N. (2012). *Science*, *336* (6084), 998. **[4]** Katsos, N., Cummins, C., Ezeizabarrena, M.-J., Gavarró, A., Kraljević, J. K., Hrzica, G., . . . , Noveck, I. (2016). *PNAS*, *113* (33), 9244–9249.