Experimental pragmatics: a probability-logical perspective

Niki Pfeifer

Munich Center for Mathematical Philosophy (LMU Munich)

Associated project:
Coherence-based probability logic: rationality under uncertainty
(DFG project PF 740/2-2 within the SPP 1516)

Table of contents

Table of contents

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$
- The conditional event $C|A$ is partially truth-functional (void, if A is false and undefined, if A is a logical contradiction)

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$
- The conditional event $C|A$ is partially truth-functional (void, if A is false and undefined, if A is a logical contradiction)
- Arguments: $\langle$ premise(s) , conclusion $\rangle$

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$
- The conditional event $C|A$ is partially truth-functional (void, if A is false and undefined, if A is a logical contradiction)
- Arguments: $\langle$ premise(s) , conclusion $\rangle$
- Premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$
- The conditional event $C|A$ is partially truth-functional (void, if A is false and undefined, if A is a logical contradiction)
- Arguments: $\langle$ premise(s) , conclusion $\rangle$
- Premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)
- **Key idea: Uncertainty is propagated deductively from the premises to the conclusion**

Mental probability logic (Pfeifer, 2006, 2012a, 2012b, 2013, 2014; Pfeifer & Kleiter, 2005)

- Uncertain indicative If A, then C is interpreted as $p(C|A)$
- The conditional event $C|A$ is partially truth-functional (void, if A is false and undefined, if A is a logical contradiction)
- Arguments: $\langle$ premise(s) , conclusion $\rangle$
- Premises contain:
  - probabilistic and/or logical information
  - background knowledge (if available)
- **Key idea: Uncertainty is propagated deductively from the premises to the conclusion**
- Rationality framework: coherence based probability logic

Coherence based probability logic

- Coherence
  - de Finetti, and $\{$Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, $\ldots\}$
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $p(B|A)$ by

$$\frac{p(A \wedge B)}{p(A)} \quad \text{and assume that} \quad p(A) > 0$$

Coherence based probability logic

- Coherence
  - de Finetti, and $\{$Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...$\}$
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $p(B|A)$ by

$$\frac{p(A \wedge B)}{p(A)} \quad \text{and assume that} \quad p(A) > 0$$

what if $p(A) = 0$?

Coherence based probability logic

- Coherence
  - de Finetti, and $\{$Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...$\}$
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $p(B|A)$ by

    $$\frac{p(A \wedge B)}{p(A)} \quad \text{and assume that} \quad p(A) > 0$$

    what if $p(A) = 0$?
    in the coherence approach, conditional probability, $p(B|A)$, is primitive
  - zero probabilities are exploited to reduce the complexity
  - imprecision

Coherence based probability logic

- Coherence
  - de Finetti, and {Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, . . . }
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $p(B|A)$ by

    $$\frac{p(A \wedge B)}{p(A)} \quad \text{and assume that} \quad p(A) > 0$$

    what if $p(A) = 0$?

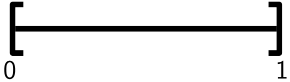
    in the coherence approach, conditional probability, $p(B|A)$, is primitive
  - zero probabilities are exploited to reduce the complexity
  - imprecision
- Probability logic
  - uncertain argument forms
  - deductive consequence relation
  - **propagation of the uncertainties from the premises to the conclusions**

Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C \mid A) = x$
A	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$

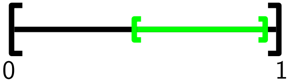
Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = x$
A	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$



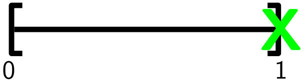
Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = .90$
A	$p(A) = .50$
C	$.45 \leq p(C) \leq .95$



Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C \mid A) = 1$
A	$p(A) = 1$
C	$p(C) = 1$



Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = 0$
A	$p(A) = 0$
C	$0 \leq p(C) \leq 1$

From probability logic to probabilisitic pragmatics

Consider a probability logical argument with n premises:

$$\frac{\begin{array}{l}\text{Premise 1}\\ \dots\\ \text{Premise n}\end{array}}{\text{Conclusion}}$$

From probability logic to probabilisitic pragmatics

Consider a probability logical argument with *n* premises:

Premise 1	$\Longrightarrow$	...what the speaker says
...	...	...
Premise n	$\Longrightarrow$	...what the speaker says
Conclusion	$\Longrightarrow$	...what the listeners hears/infers

Why not classical logic?

Truth-functional bivalent logic is

- unable to deal with degrees of belief
- unable to deal with nonmonotonicity
- interpreting natural language conditionals by the material conditional ($A \supset C$, which is logically equivalent to $\neg A \vee C$) is highly problematic

Table of contents

Paradoxes of the material conditional

(Paradox 1)	(Paradox 2)
B	Not: A
If A, then B	If A, then B

Paradoxes of the material conditional

(Paradox 1)
$$\frac{B}{\text{If } A\text{, then } B}$$

(Paradox 2)
$$\frac{\text{Not: } A}{\text{If } A\text{, then } B}$$

(Paradox 1)
$$\frac{B}{A \supset B}$$

(Paradox 2)
$$\frac{\neg A}{A \supset B}$$

Sample paradoxes of the material conditional

(Paradox 1)

$$\frac{p(B) = x}{x \leq p(A \supset B) \leq 1}$$

(Paradox 2)

$$\frac{p(\neg A) = x}{1 - x \leq p(A \supset B) \leq 1}$$

probabilistically informative

Sample paradoxes of the material conditional (Pfeifer, 2014)

Paradoxes of the material conditional, e.g.,

(Paradox 1)
$$\frac{p(B) = x}{0 \leq p(B|A) \leq 1}$$

(Paradox 2)
$$\frac{p(\neg A) = x}{0 \leq p(B|A) \leq 1}$$

probabilistically non-informative

Sample paradoxes of the material conditional (Pfeifer, 2014)

Paradoxes of the material conditional, e.g.,

(Paradox 1)

$$\frac{p(B) = x}{0 \leq p(B|A) \leq 1}$$

(Paradox 2)

$$\frac{p(\neg A) = x}{0 \leq p(B|A) \leq 1}$$

probabilistically non-informative

This matches the data (Pfeifer & Kleiter, 2011).

Sample paradoxes of the material conditional (Pfeifer, 2014)

Paradoxes of the material conditional, e.g.,

(Paradox 1)

$$\frac{p(B) = x}{0 \leq p(B|A) \leq 1}$$

(Paradox 2)

$$\frac{p(\neg A) = x}{0 \leq p(B|A) \leq 1}$$

probabilistically non-informative

This matches the data (Pfeifer & Kleiter, 2011).

Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:

If $p(B) = 1$, then $p(A \wedge B) = p(A)$.

Sample paradoxes of the material conditional (Pfeifer, 2014)

Paradoxes of the material conditional, e.g.,

(Paradox 1)

$$\frac{p(B) = x}{0 \leq p(B|A) \leq 1}$$

(Paradox 2)

$$\frac{p(\neg A) = x}{0 \leq p(B|A) \leq 1}$$

probabilistically <u>non</u>-informative

This matches the data (Pfeifer & Kleiter, 2011).

Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:

If $p(B) = 1$, then $p(A \wedge B) = p(A)$. Thus,

$p(B|A) = \frac{p(A \wedge B)}{p(A)} = \frac{p(A)}{p(A)} = 1$, if $p(A) > 0$.

Informat. vers. of the paradoxes (Pfeifer, 2014; Pfeifer & Douven, 2014)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

Informat. vers. of the paradoxes (Pfeifer, 2014; Pfeifer & Douven, 2014)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \supset B \equiv \top$ infer $\Pr(B \mid A) = 1$ is coherent.

Informat. vers. of the paradoxes (Pfeifer, 2014; Pfeifer & Douven, 2014)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \supset B \equiv \top$ infer $\Pr(B \mid A) = 1$ is coherent.

From $\Pr(B) = x$ and $\Pr(A) = y$ infer

$$\max\left\{0, \frac{x + y - 1}{y}\right\} \leqslant \Pr(B \mid A) \leqslant \min\left\{\frac{x}{y}, 1\right\} \text{ is coherent.}$$

Informat. vers. of the paradoxes (Pfeifer, 2014; Pfeifer & Douven, 2014)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

From $\Pr(B) = 1$ and $A \supset B \equiv \top$ infer $\Pr(B \mid A) = 1$ is coherent.

From $\Pr(B) = x$ and $\Pr(A) = y$ infer

$$\max\left\{0, \frac{x + y - 1}{y}\right\} \leqslant \Pr(B \mid A) \leqslant \min\left\{\frac{x}{y}, 1\right\} \text{ is coherent.}$$

...a special case of the cautious monotonicity rule of System P (Gilio, 2002).

Truth tables

Negation:

A	not-A $\neg A$
T	F
F	T

Samples of other connectives:

A	B	A and B $A \wedge B$	A or B $A \vee B$	If A, then B $A \supset B$	A iff B $A \equiv B$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Truth tables & Ramsey test

Negation:

A	not-A $\neg A$
T	F
F	T

Samples of other connectives:

A	B	A and B $A \wedge B$	A or B $A \vee B$	If A, then B $A \supset B$	A iff B $A \equiv B$	B given A $B \vert A$
T	T	T	T	T	T	T
T	F	F	T	F	F	F
F	T	F	T	T	F	void
F	F	F	F	T	T	void

"If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; ...We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered void" (Ramsey, 1929/1994, footnote, p. 155).

Truth tables & Ramsey test

Samples of other connectives:

A	B			If A, then B $A \supset B$		B given A $B \mid A$
T	T			T		T
T	F			F		F
F	T			T		void
F	F			T		void

> *"If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; . . . We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered void"* (Ramsey, 1929/1994, footnote, p. 155).

Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
p(A \wedge C) & = & x_1 \\
p(A \wedge \neg C) & = & x_2 \\
p(\neg A \wedge C) & = & x_3 \\
p(\neg A \wedge \neg C) & = & x_4 \\
\hline
p(\text{If } A, \text{ then } C) & = & ?
\end{array}
$$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 \\ p(A \wedge \neg C) & = & x_2 \\ p(\neg A \wedge C) & = & x_3 \\ p(\neg A \wedge \neg C) & = & x_4 \\ \hline p(\text{If } A \text{, then } C) & = & ? \end{array}$$

Sample conclusion candidates:

- $p(A \wedge C) = x_1$
- $p(C|A) = x_1/(x_1 + x_2)$
- $p(A \supset C) = x_1 + x_3 + x_4$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 = .25 \\ p(A \wedge \neg C) & = & x_2 = .25 \\ p(\neg A \wedge C) & = & x_3 = .25 \\ p(\neg A \wedge \neg C) & = & x_4 = .25 \\ \hline p(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Sample conclusion candidates:

- $p(A \wedge C) = x_1$
- $p(C|A) = x_1/(x_1 + x_2)$
- $p(A \supset C) = x_1 + x_3 + x_4$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
p(A \wedge C) & = & x_1 = .25 \\
p(A \wedge \neg C) & = & x_2 = .25 \\
p(\neg A \wedge C) & = & x_3 = .25 \\
p(\neg A \wedge \neg C) & = & x_4 = .25 \\
\hline
p(\text{If } A, \text{ then } C) & = & ?
\end{array}
$$

Sample conclusion candidates:

- $p(A \wedge C) = x_1 = .25$
- $p(C|A) = x_1/(x_1 + x_2) = .50$
- $p(A \supset C) = x_1 + x_3 + x_4 = .75$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 \\ p(A \wedge \neg C) & = & x_2 \\ p(\neg A \wedge C) & = & x_3 \\ p(\neg A \wedge \neg C) & = & x_4 \\ \hline p(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Main results:

- more than half of the responses are consistent with $p(C|A)$
- many responses are consistent with $p(A \wedge C)$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 \\ p(A \wedge \neg C) & = & x_2 \\ p(\neg A \wedge C) & = & x_3 \\ p(\neg A \wedge \neg C) & = & x_4 \\ \hline p(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Main results:

- more than half of the responses are consistent with $p(C|A)$
- many responses are consistent with $p(A \wedge C)$
- Iterated version: interpretation shifts to $p(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011)

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} p(A \wedge C) & = & x_1 \\ p(A \wedge \neg C) & = & x_2 \\ p(\neg A \wedge C) & = & x_3 \\ p(\neg A \wedge \neg C) & = & x_4 \\ \hline p(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Main results:

- more than half of the responses are consistent with $p(C|A)$
- many responses are consistent with $p(A \wedge C)$
- Iterated version: interpretation shifts to $p(C|A)$ (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011)

Key feature:

- reasoning under complete probabilistic knowledge

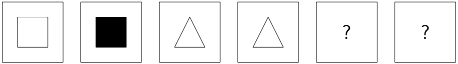
Experiment

Motivation

- probabilistic truth table task with incomplete probabilistic knowledge
- Is the conditional event interpretation still dominant?
- Are there shifts of interpretation?

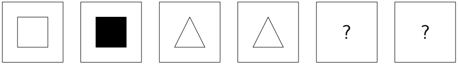
Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Example: Task 5 (Pfeifer, 2013)

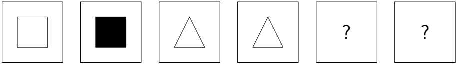
Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



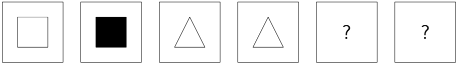
Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

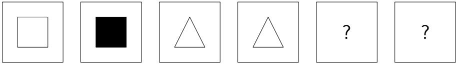
Answer:

at least	*at most*
☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6
out of ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	out of ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

(please tick the appropriate boxes)

Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Cond. event: *at least* 1 out of 5 and *at most* 3 out of 5

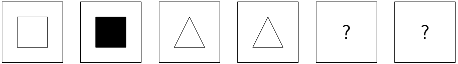
at least							
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

at most							
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

(please tick the appropriate boxes)

Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Conjunction: *at least* 1 out of 6 and *at most* 3 out of 6

at least

☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6
out of	☐	☐	☐	☐	☐	☐
	1	2	3	4	5	6

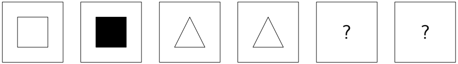
at most

☐	☐	☐	☐	☐	☐	☐
0	1	2	3	4	5	6
out of	☐	☐	☐	☐	☐	☐
	1	2	3	4	5	6

(please tick the appropriate boxes)

Example: Task 5 (Pfeifer, 2013)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Mat. cond.: *at least* 2 out of 6 and *at most* 4 out of 6

at least

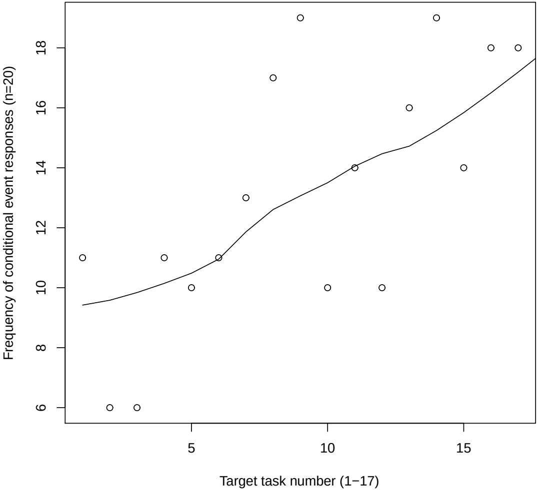
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

at most

	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

(please tick the appropriate boxes)

Results (n = 20) (Pfeifer, 2013)



shift of conditional probability responses from 38.3% (first three) to 83.3% (last three tasks)

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017)

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as . . .
- . . . a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ... a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ... a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)
- data are also robust for "missing link" conditionals, i.e., conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

... evidence against the inferentialist account of conditionals

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ... a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)
- data are also robust for "missing link" conditionals, i.e., conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

  ... evidence against the inferentialist account of conditionals
- "experts": 80% conditional probability responses and no shifts

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ... a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)
- data are also robust for "missing link" conditionals, i.e., conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

  ... evidence against the inferentialist account of conditionals
- "experts": 80% conditional probability responses and no shifts
- apparent pragmatic/relevance effect when conditionals are "packed" (e.g., "If the card shows a 2, then the card shows an even number") or "unpacked" ("If the card shows a 2, then the card shows a 2 or a 4")

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ...a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)
- data are also robust for "missing link" conditionals, i.e., conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

  ...evidence against the inferentialist account of conditionals
- "experts": 80% conditional probability responses and no shifts
- apparent pragmatic/relevance effect when conditionals are "packed" (e.g., "If the card shows a 2, then the card shows an even number") or "unpacked" ("If the card shows a 2, then the card shows a 2 or a 4"):
  Most people judge (correctly) $p(even|x = 2) = 1$

Further results

$p(C|A)$ best predictor for beliefs in conditionals, even if

- $x_1, \ldots, x_4$ is precise or imprecise (Pfeifer, 2013)
- the conditional is formulated as a causal conditional (Over et al., 2007; Pfeifer & Stöckle-Schobel, 2015) or as an abductive conditional (Pfeifer & Tulkki, 2017) or as ...
- ...a counterfactual, formulated by a fact (not A) and a conditional in subjunctive mood *If A were the case, C would be the case* (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017)
- evidence supports the universality hypothesis (Pfeifer & Yama, 2017)
- data are also robust for "missing link" conditionals, i.e., conditionals which violate Δp:

$$\Delta p = p(C|A) - p(C|\neg A) > 0$$

  ...evidence against the inferentialist account of conditionals
- "experts": 80% conditional probability responses and no shifts
- apparent pragmatic/relevance effect when conditionals are "packed" (e.g., "If the card shows a 2, then the card shows an even number") or "unpacked" ("If the card shows a 2, then the card shows a 2 or a 4"):
  Most people judge (correctly) $p(even|x = 2) = 1$
  but (incorrectly) $p(x = 2 \vee x = 4|x = 2) = 0$ (Fugard, Pfeifer, & Mayerhofer, 2011)

From modus ponens to generalised modus ponens

	Modus ponens	Generalised modus ponens
(Categorical premise)	A	$A\|H$
(Conditional premise)	If A, then C	If $A\|H$, then C
(Conclusion)	C	C

From modus ponens to generalised modus ponens

	Modus ponens	Generalised modus ponens
(Categorical premise)	A	$A \| H$
(Conditional premise)	If A, then C	If $A \| H$, then C
(Conclusion)	C	C

Sample instantiation (Gibbard, 1981, p. 237):

$\overbrace{\textit{The cup breaks if dropped}}^{A|H}$.

If $\overbrace{\textit{the cup breaks if dropped}}^{A|H}$, then $\overbrace{\textit{the cup is fragile}}^{C}$.

Therefore, $\overbrace{\textit{the cup is fragile}}^{C}$.

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A \vert H$	$p(A \vert H) = x$
If $A \vert H$, then C	$\mathbb{P}[C \vert (A \vert H)] = y$
C	$? \leq p(C) \leq ?$

What does the conditional premise mean? It is a conditional random quantity.

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A \vert H$	$p(A \vert H) = x$
If $A \vert H$, then C	$\mathbb{P}[C \vert (A \vert H)] = y$
C	$? \leq p(C) \leq ?$

What does the conditional premise mean? It is a conditional random quantity.

How can we assess its uncertainty? By its prevision (denoted by $\mathbb{P}$).

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

What does the conditional premise mean? It is a conditional random quantity.

How can we assess its uncertainty? By its prevision (denoted by $\mathbb{P}$).

In betting terms, $\mu = \mathbb{P}[C|(A|H)]$ represents the amount you agree to pay, with the proviso that you will receive the quantity:

$$C|(A|H) = \begin{cases} 1, & \text{if } A \wedge H \wedge C \text{ true,} \\ 0, & \text{if } A \wedge H \wedge \neg C \text{ true,} \\ \mu, & \text{if } \neg A \wedge H \text{ true,} \\ x + \mu(1-x), & \text{if } \neg H \wedge C \text{ true,} \\ \mu(1-x), & \text{if } \neg H \wedge \neg C \text{ true.} \end{cases}$$

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

What does the conditional premise mean? It is a conditional random quantity.

How can we assess its uncertainty? By its prevision (denoted by $\mathbb{P}$).

In betting terms, $\mu = \mathbb{P}[C|(A|H)]$ represents the amount you agree to pay, with the proviso that you will receive the quantity:

$$C|(A|H) = \begin{cases} 1, & \text{if } A \wedge H \wedge C \text{ true}, \\ 0, & \text{if } A \wedge H \wedge \neg C \text{ true}, \\ \mu, & \text{if } \neg A \wedge H \text{ true}, \\ x + \mu(1-x), & \text{if } \neg H \wedge C \text{ true}, \\ \mu(1-x), & \text{if } \neg H \wedge \neg C \text{ true}. \end{cases}$$

Since $(C|A)|H \neq C|(A \wedge H)$, the Import-Export Principle does not hold. Thus, Lewis' first triviality result (1976) is avoided (Gilio & Sanfilippo, 2014).

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A \mid H$	$p(A \mid H) = x$
If $A \mid H$, then C	$\mathbb{P}[C \mid (A \mid H)] = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Theorem

Given any coherent assessment (x, y) on $\{A \mid H, C \mid (A \mid H)\}$, with A, C, H logically independent, but $A \neq \bot$ and $H \neq \bot$. The conclusion $p(C)$ is coherent iff $xy \leq p(C) \leq xy + 1 - x$

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \bot$ and $H \neq \bot$. The conclusion $p(C)$ is coherent iff $xy \leq p(C) \leq xy + 1 - x$,

which are just the same probability propagation rules as in the non-nested probabilistic modus ponens (i.e., from $p(A) = x$ and $p(C|A) = y$ infer $xy \leq p(C) \leq xy + 1 - x$).

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}[C\|(A\|H)] = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \bot$ and $H \neq \bot$. The conclusion $p(C)$ is coherent iff $xy \leq p(C) \leq xy + 1 - x$,

which are just the same probability propagation rules as in the non-nested probabilistic modus ponens (i.e., from $p(A) = x$ and $p(C|A) = y$ infer $xy \leq p(C) \leq xy + 1 - x$). Similar results: Centering and Conjunction (Gilio, Over, Pfeifer, & Sanfilippo, 2017; Sanfilippo, Pfeifer, Over, & Gilio, 2018).

Further examples (Sanfilippo, Pfeifer, Over, & Gilio, 2018, *Int. Jour. of Approx. Reasoning*)

Conjunction:

Nested (Theorem 1)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(A|H) \wedge (B|K)]$ is...

Non-nested (Fréchet-Hoeffding bounds)

from $p(A) = x$ and $p(B) = y$

infer $p(A \wedge B)$ is...

Further examples (Sanfilippo, Pfeifer, Over, & Gilio, 2018, *Int. Jour. of Approx. Reasoning*)

Conjunction:

Nested (Theorem 1)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(A|H) \wedge (B|K)]$ is...

Non-nested (Fréchet-Hoeffding bounds)

from $p(A) = x$ and $p(B) = y$

infer $p(A \wedge B)$ is...

... *at least* $\max\{x + y - 1, 0\}$ and *at most* $\min\{x, y\}$

Further examples (Sanfilippo, Pfeifer, Over, & Gilio, 2018, *Int. Jour. of Approx. Reasoning*)

Conjunction:

Nested (Theorem 1)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(A|H) \wedge (B|K)]$ is...

Non-nested (Fréchet-Hoeffding bounds)

from $p(A) = x$ and $p(B) = y$

infer $p(A \wedge B)$ is...

... *at least* $\max\{x + y - 1, 0\}$ and *at most* $\min\{x, y\}$

Centering:

Nested (Theorem 4)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(B|K)|(A|H)]$ is...

Non-nested (Theorem 6)

from $p(A) = x$ and $p(B) = y$

infer $p(B|A)$ is...

Further examples (Sanfilippo, Pfeifer, Over, & Gilio, 2018, *Int. Jour. of Approx. Reasoning*)

Conjunction:

Nested (Theorem 1)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(A|H) \wedge (B|K)]$ is...

Non-nested (Fréchet-Hoeffding bounds)

from $p(A) = x$ and $p(B) = y$

infer $p(A \wedge B)$ is...

... *at least* $\max\{x + y - 1, 0\}$ and *at most* $\min\{x, y\}$

Centering:

Nested (Theorem 4)

from $p(A|H) = x$ and $p(B|K) = y$

infer $\mathbb{P}[(B|K)|(A|H)]$ is...

Non-nested (Theorem 6)

from $p(A) = x$ and $p(B) = y$

infer $p(B|A)$ is...

... *at least* $\begin{cases} \max\left\{\frac{x+y-1}{x}, 0\right\}, & \text{if } x > 0; \\ 0, & \text{if } x = 0; \end{cases}$ and *at most* $\begin{cases} \min\left\{\frac{y}{x}, 1\right\}, & \text{if } x > 0; \\ 1, & \text{if } x = 0. \end{cases}$

Recall: results of the probabilistic truth table tasks

Most people interpret their beliefs in counterfactuals by $p(C|A)$,

Recall: results of the probabilistic truth table tasks

Most people interpret their beliefs in counterfactuals by $p(C|A)$, i.e., their degree of belief in *If A were the case, C would be the case* equals $p(C|A)$

(see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017).

Recall: results of the probabilistic truth table tasks

Most people interpret their beliefs in counterfactuals by $p(C|A)$, i.e., their degree of belief in *If A were the case, C would be the case* equals $p(C|A)$ (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017).

Why does conditional probability predict counterfactuals?

Recall: results of the probabilistic truth table tasks

Most people interpret their beliefs in counterfactuals by $p(C|A)$, i.e., their degree of belief in *If A were the case, C would be the case* equals $p(C|A)$ (see, e.g., Pfeifer, 2013; Pfeifer & Stöckle-Schobel, 2015; Pfeifer & Tulkki, 2017).

Why does conditional probability predict counterfactuals?

Formally (see, e.g. Gilio & Sanfilippo, 2013),

$$\overbrace{\underbrace{\textit{Prevision}\quad [(C|A)\,|\,\overbrace{\neg A}^{\text{fact}}]}_{\text{cond. random quantity}}}^{\text{belief in counterfactual}} \quad = \quad \overbrace{\textit{Probability}\;(\underbrace{C|\;\overbrace{A}^{\text{assumed}}}_{cond. event})}^{\text{belief in indicative conditional}}\;.$$

Table of contents

The traditional square of opposition

INSERT GRAPHICS HERE

(attributed to Apuleius of Madaura (2^{nd} century CE);

Schoenberg collection, manuscript from Fleury Abbey, approx. 9^{th} century)

- Aristotle distinguished contradiction and contrariety
- diagram appeared in the 2^{nd} century CE (e.g., Boethius)
- popular in medieval times
- recently revisited (see, e.g. Beziau & Read, 2014; Dubois & Prade, 2012)

The traditional square of opposition

INSERT GRAPHICS HERE

(attributed to Apuleius of Madaura (2nd century CE);

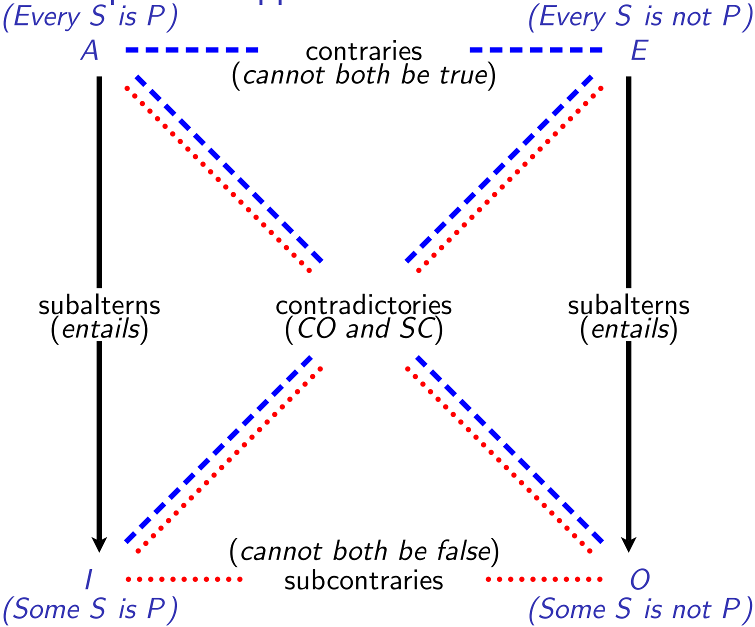
Schoenberg collection, manuscript from Fleury Abbey, approx. 9th century)

- Aristotle distinguished contradiction and contrariety
- diagram appeared in the 2nd century CE (e.g., Boethius)
- popular in medieval times
- recently revisited (see, e.g. Beziau & Read, 2014; Dubois & Prade, 2012)
- Motivation: to build a psychologically plausible rationality framework for relations among basic sentence types

Basic sentences types A, E, I, O (Pfeifer & Sanfilippo, 2017)

Sentence type	
universal affirmative (A)	Every S is P (All S are P)
particular affirmative (I)	Some S is P (At least one S is P)
universal negative (E)	Every S is not P (No S are P)
particular negative (O)	Some S is not P (At least one S is not P)

Traditional square of opposition



Probabilistic interpretation of the basic sentences A, E, I, O

(Pfeifer & Sanfilippo, 2017)

Sentence $\mathcal{S}$	Prob. interpretation	Assessment on $P\|S$
A : Every S is P	$p(P\|S) = 1$	$\mathcal{I}_A = \{1\}$
E : Every S is not P	$p(\neg P\|S) = 1$	$\mathcal{I}_E = \{0\}$
I : Some S is P	$p(P\|S) > 0$	$\mathcal{I}_I =]0, 1]$
O : Some S is not P	$p(\neg P\|S) > 0$	$\mathcal{I}_O = [0, 1[$

Generalised quantifiers (Pfeifer & Sanfilippo, 2017)

Probabilistic interpretation of the sentence types A, E, I, and O involving generalised quantifiers Q, where x denotes a threshold (with $x \in]\frac{1}{2}, 1]$)

Generalised quantifiers (Pfeifer & Sanfilippo, 2017)

Probabilistic interpretation of the sentence types A, E, I, and O involving generalised quantifiers Q, where x denotes a threshold (with $x \in]\frac{1}{2}, 1]$)

Sentence		Probability constraints	Assessment on $P|S$
$A(x):$	$(Q_{\geq x}$ S are $P)$	$p(P|S) \geq x$	$\mathcal{I}_{A(x)} = [x, 1]$
$E(x):$	$(Q_{\geq x}$ S are not $P)$	$p(\neg P|S) \geq x$	$\mathcal{I}_{E(x)} = [0, 1-x]$
$I(x):$	$(Q_{>1-x}$ S are $P)$	$p(P|S) > 1-x$	$\mathcal{I}_{I(x)} =]1-x, 1]$
$O(x):$	$(Q_{>1-x}$ S are not $P)$	$p(\neg P|S) > 1-x$	$\mathcal{I}_{O(x)} = [0, x[$

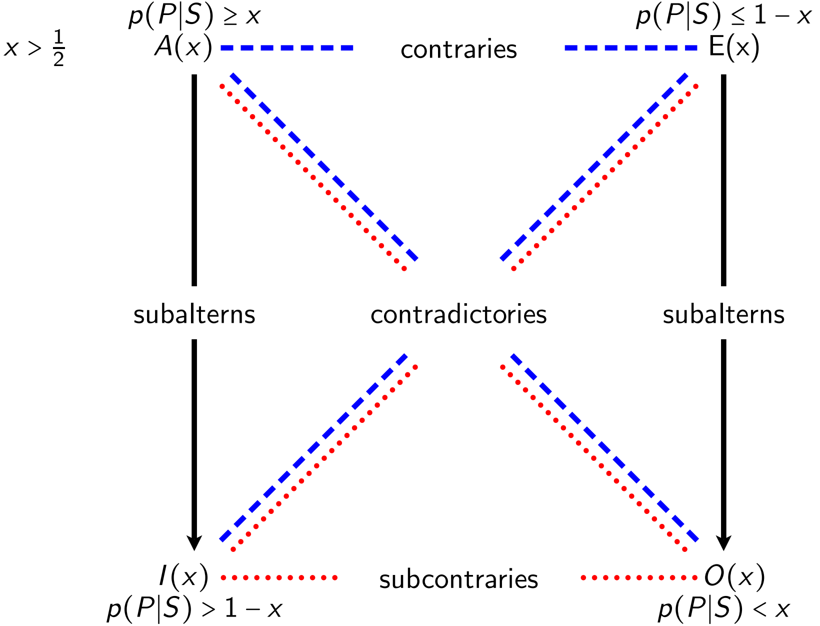
Generalised quantifiers (Pfeifer & Sanfilippo, 2017)

Probabilistic interpretation of the sentence types A, E, I, and O involving generalised quantifiers Q, where x denotes a threshold (with $x \in]\frac{1}{2}, 1]$)

Sentence		Probability constraints	Assessment on $P\|S$
$A(x):$	$(Q_{\geq x}$ S are $P)$	$p(P\|S) \geq x$	$\mathcal{I}_{A(x)} = [x, 1]$
$E(x):$	$(Q_{\geq x}$ S are not $P)$	$p(\neg P\|S) \geq x$	$\mathcal{I}_{E(x)} = [0, 1-x]$
$I(x):$	$(Q_{>1-x}$ S are $P)$	$p(P\|S) > 1-x$	$\mathcal{I}_{I(x)} =]1-x, 1]$
$O(x):$	$(Q_{>1-x}$ S are not $P)$	$p(\neg P\|S) > 1-x$	$\mathcal{I}_{O(x)} = [0, x[$

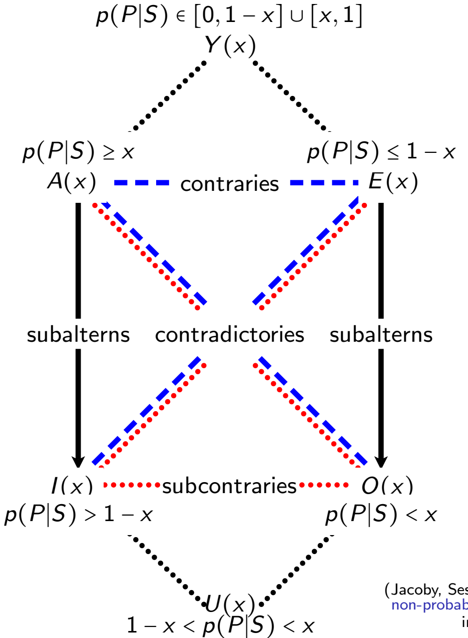
- the basic syllogistic sentence types (A, E, I, O) correspond to the extreme case $x = 1$
- x may be context dependent
- allows for modeling various linguistic expressions like "most," "almost all," etc.

Square of opposition & general'd quantifiers (Pfeifer & Sanfilippo, 2017)



Probabilistic hexagon of opposition (Pfeifer & Sanfilippo, 2017)

generalised quantifiers with $x > \frac{1}{2}$



(Jacoby, Sesmat, & Blanché proposed the non-probabilistic hexagon independently in the early 1950$^{\text{ies}}$)

Table of contents

Table of contents

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)
- most people negate indicative and counterfactual conditionals by the narrow scope reading (Pfeifer & Tulkki, 2017)

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)
- most people negate indicative and counterfactual conditionals by the narrow scope reading (Pfeifer & Tulkki, 2017)
- some alleged pragmatic effects can be explained in probability semantical terms (paradoxes)

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)
- most people negate indicative and counterfactual conditionals by the narrow scope reading (Pfeifer & Tulkki, 2017)
- some alleged pragmatic effects can be explained in probability semantical terms (paradoxes)
- theory of nested conditionals explains responses to counterfactuals in the probabilistic truth table tasks

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)
- most people negate indicative and counterfactual conditionals by the narrow scope reading (Pfeifer & Tulkki, 2017)
- some alleged pragmatic effects can be explained in probability semantical terms (paradoxes)
- theory of nested conditionals explains responses to counterfactuals in the probabilistic truth table tasks
- probabilistic semantics for categorical syllogisms with very weak existential import assumptions (e.g., $p(S|S \vee M) > 0$): Figure I (transitive structure; (Gilio, Pfeifer, & Sanfilippo, 2016)), Figure II (Pfeifer & Sanfilippo, submitted), and Figure III (in preparation).

Concluding remarks

- the probabilistic square and hexagon of opposition (Pfeifer & Sanfilippo, 2017) can serve as a new rationality framework for psychological reasoning research
- a conditional "*if A, then C*", which holds with at least 90% probability (i.e., $.9 \leq p(C|A) \leq 1$) can be negated by the
  - **narrow** scope reading ($0 \leq p(C|A) \leq .1$)
  - **wide** scope reading ($0 \leq p(C|A) < .9$)
- most people negate indicative and counterfactual conditionals by the narrow scope reading (Pfeifer & Tulkki, 2017)
- some alleged pragmatic effects can be explained in probability semantical terms (paradoxes)
- theory of nested conditionals explains responses to counterfactuals in the probabilistic truth table tasks
- probabilistic semantics for categorical syllogisms with very weak existential import assumptions (e.g., $p(S|S \vee M) > 0$): Figure I (transitive structure; (Gilio, Pfeifer, & Sanfilippo, 2016)), Figure II (Pfeifer & Sanfilippo, submitted), and Figure III (in preparation).

Papers available at: `pfeifer.userweb.mwn.de`

References I

Beziau, J.-Y., & Read, S. (2014). Editorial: Square of opposition: A diagram and a theory in historical perspective. *History and Philosophy of Logic*, *35*(4), 315–316.

Dubois, D., & Prade, H. (2012). From Blanché's hexagonal organization of concepts to formal concept analysis and possibility theory. *Logica Universalis*, *6*(1-2), 149–169. Retrieved from `http://dx.doi.org/10.1007/s11787-011-0039-0` doi: 10.1007/s11787-011-0039-0

Evans, J. St. B. T., Handley, S. J., & Over, D. E. (2003). Conditionals and conditional probability. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *29*(2), 321-355.

References II

Fugard, A. J. B., Pfeifer, N., & Mayerhofer, B. (2011). Probabilistic theories of reasoning need pragmatics too: Modulating relevance in uncertain conditionals. *Journal of Pragmatics*, *43*, 2034-2042.

Fugard, A. J. B., Pfeifer, N., Mayerhofer, B., & Kleiter, G. D. (2011). How people interpret conditionals: Shifts towards the conditional event. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *37*(3), 635–648.

Gibbard, A. (1981). Two recent theories of conditionals. In W. L. Harper, R. Stalnaker, & G. Pearce (Eds.), *Ifs* (pp. 221–247). Dordrecht: Reidel.

Gilio, A. (2002). Probabilistic reasoning under coherence in System P. *Annals of Mathematics and Artificial Intelligence*, *34*, 5-34.

References III

Gilio, A., Over, D. E., Pfeifer, N., & Sanfilippo, G. (2017). Centering and compound conditionals under coherence. In M. B. Ferraro et al. (Eds.), *Soft methods for data science* (pp. 253–260). Berlin, Heidelberg: Springer.

Gilio, A., Pfeifer, N., & Sanfilippo, G. (2016). Transitivity in coherence-based probability logic. *Journal of Applied Logic*, *14*, 46–64. doi: http://dx.doi.org/10.1016/j.jal.2015.09.012

Gilio, A., & Sanfilippo, G. (2013). Conditional random quantities and iterated conditioning in the setting of coherence. In L. C. van der Gaag (Ed.), *ECSQARU 2013* (Vol. 7958, pp. 218–229). Berlin, Heidelberg: Springer.

Gilio, A., & Sanfilippo, G. (2014). Conditional random quantities and compounds of conditionals. *Studia Logica*, *102*(4), 709-729.

Jackson, F. (Ed.). (1991). *Conditionals*. Oxford: Oxford University Press.

References IV

Lewis, D. (1976). Probabilities of conditionals and conditional probabilities. *Philosophical Review*, *85*, 297-315. (Reprint with postscript in (Jackson, 1991, 76–101); the page references are to the reprint)

Oberauer, K., & Wilhelm, O. (2003). The meaning(s) of conditionals: Conditional probabilities, mental models and personal utilities. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *29*(4), 680–693.

Over, D. E., Hadjichristidis, C., Evans, J. St. B. T., Handley, S. J., & Sloman, S. (2007). The probability of causal conditionals. *Cognitive Psychology*, *54*, 62-97.

Pfeifer, N. (2006). *On mental probability logic* (Unpublished doctoral dissertation). Department of Psychology, University of Salzburg. (The abstract is published in *The Knowledge Engineering Review*, 2008, 23, 217–226)

References V

Pfeifer, N. (2012a). Experiments on Aristotle's Thesis: Towards an experimental philosophy of conditionals. *The Monist*, *95*(2), 223–240.

Pfeifer, N. (2012b). *Naturalized formal epistemology of uncertain reasoning* (Unpublished doctoral dissertation). Tilburg Center for Logic and Philosophy of Science, Tilburg University.

Pfeifer, N. (2013). The new psychology of reasoning: A mental probability logical perspective. *Thinking & Reasoning*, *19*(3–4), 329–345.

Pfeifer, N. (2014). Reasoning about uncertain conditionals. *Studia Logica*, *102*(4), 849–866.

Pfeifer, N., & Douven, I. (2014). Formal epistemology and the new paradigm psychology of reasoning. *The Review of Philosophy and Psychology*, *5*(2), 199–221.

Pfeifer, N., & Kleiter, G. D. (2005). Towards a mental probability logic. *Psychologica Belgica*, *45*(1), 71-99.

References VI

Pfeifer, N., & Kleiter, G. D. (2010). The conditional in mental probability logic. In M. Oaksford & N. Chater (Eds.), *Cognition and conditionals: Probability and logic in human thought* (pp. 153–173). Oxford: Oxford University Press.

Pfeifer, N., & Kleiter, G. D. (2011). Uncertain deductive reasoning. In K. Manktelow, D. E. Over, & S. Elqayam (Eds.), *The science of reason: A Festschrift for Jonathan St. B.T. Evans* (p. 145-166). Hove: Psychology Press.

Pfeifer, N., & Sanfilippo, G. (2017). Probabilistic squares and hexagons of opposition under coherence. *International Journal of Approximate Reasoning*, *88*, 282–294.

Pfeifer, N., & Sanfilippo, G. (submitted). *Probabilistic semantics for categorical syllogisms of Figure II.*

References VII

Pfeifer, N., & Stöckle-Schobel, R. (2015). Uncertain conditionals and counterfactuals in (non-)causal settings. In G. Arienti, B. G. Bara, & S. G. (Eds.), *Proceedings of the EuroAsianPacific Joint Conference on Cognitive Science (4th European Conference on Cognitive Science; 10th International Conference on Cognitive Science)* (Vol. 1419, pp. 651–656). Aachen: CEUR Workshop Proceedings. Retrieved from http://ceur-ws.org/Vol-1419/paper0108.pdf

Pfeifer, N., & Tulkki, L. (2017). Abductive, causal, and counterfactual conditionals under incomplete probabilistic knowledge. In G. Gunzelmann, A. Howes, T. Tenbrink, & E. Davelaar (Eds.), *Proceedings of the 39th Cognitive Science Society Meeting* (pp. 2888–2893). Austin, TX: The Cognitive Science Society.

References VIII

Pfeifer, N., & Tulkki, L. (2017). Conditionals, counterfactuals, and rational reasoning. An experimental study on basic principles. *Minds and Machines*, *27*(1), 119–165.

Pfeifer, N., & Yama, H. (2017). Counterfactuals, indicative conditionals, and negation under uncertainty: Are there cross-cultural differences? In G. Gunzelmann, A. Howes, T. Tenbrink, & E. Davelaar (Eds.), *Proceedings of the 39th Cognitive Science Society Meeting* (pp. 2882–2887). Austin, TX: The Cognitive Science Society.

Ramsey, F. P. (1929/1994). General propositions and causality (1929). In D. H. Mellor (Ed.), *Philosophical papers by F. P. Ramsey* (p. 145-163). Cambridge: Cambridge University Press.

Sanfilippo, G., Pfeifer, N., & Gilio, A. (2017). Generalized probabilistic modus ponens. In A. Antonucci, C. L., & O. Papini (Eds.), *Lecture notes LNAI* (Vol. 10369, pp. 480–490). Dordrecht: Springer.

References IX

Sanfilippo, G., Pfeifer, N., Over, D. E., & Gilio, A. (2018). Probabilistic inferences from conjoined to iterated conditionals. *International Journal of Approximate Reasoning*, *93*, 103–118.